Probability and Statistic Module

Lesson 1 — Introduction to Probability

Definition of Probability

Probability is a measure of how likely an event is to occur.

P(event) = (number of favorable outcomes) / **(**total number of possible outcomes)

Probability is always between 0 (impossible) and 1 (certain). It can be written as a fraction, decimal, or percent.

Theoretical vs. Experimental Probability

Theoretical Probability	**Experimental Probability**
Based on what should happen mathematically — no experiment needed.	Based on what actually happens when you conduct an experiment or collect data.
Example: Rolling a 3 on a standard die	**Example:** Rolling a die 60 times and recording how often a 3 appears
P(3) = 1/6 ≈ 0.167 ≈ 16.7%	**If 3 appeared 12 times: P(3) = 12/60 = 1/5 = 20%**

Basic Practice

Find each probability. Write your answer as a fraction, decimal, and percent.

1. A bag contains 4 red marbles, 3 blue marbles, and 5 green marbles. One marble is drawn at random.

a) P(red) = _____ b) P(blue) = _____ c) P(green) = _____

d) P(not red) = _____ e) P(red or blue) = _____ f) P(purple) = _____

2. A standard deck of 52 cards is shuffled. One card is drawn.

a) P(heart) = _____ b) P(face card) = _____ c) P(red card) = _____

d) P(ace) = _____ e) P(red ace) = _____ f) P(not a heart) = _____

3. A letter is chosen at random from the word MATHEMATICS.

a) P(M) = _____ b) P(vowel) = _____ c) P(consonant) = _____ d) P(T) = _____

M&M Activity — Experimental Probability

Open your M&M package. Count and record each color in the table below.

M&M Color	Frequency	P(color)
Green		
Red		
Blue		
Yellow		
Brown		
Orange		
Total		

Now collect data from all groups and combine totals below.

Color	Gr.1	Gr.2	Gr.3	Gr.4	Gr.5	Gr.6	Gr.7	Gr.8	Total	P(color)
Green										
Red										
Blue										
Yellow										
Brown										
Orange										
Totals										

Discussion Questions:

1. How did your bag's experimental probabilities compare to the class totals?

_____

_____

2. Mars advertises: Brown 30%, Yellow 20%, Red 20%, Orange 10%, Green 10%, Blue 10%. How close was your class?

_____

_____

3. If you opened a new bag of 50 M&Ms, how many of each color would you predict? Explain.

_____

_____

Lesson 2 — Geometric Probability

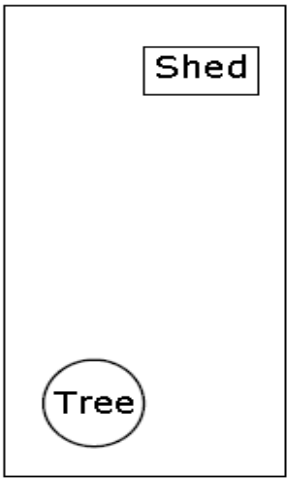
Key Idea

When outcomes correspond to lengths, areas, or regions, probability is calculated using geometric measures.

P(event) = (length / area of favorable region) / (length / area of total region)

Part A — Field, Shed, and Tree

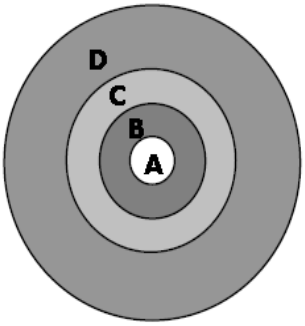
A rectangular field measures 27 feet by 15 feet. A shed and an oak tree are on the field as shown. Round all probabilities to the nearest tenth of a percent.



1. Find the area of the rectangular field.	**2.** The shed measures 5 ft by 7 ft. Find its area.
3. The oak tree's branches form a circle with diameter 6 ft. Find the area of the circle. (Use $A = \pi r^2$)	**4.** What is the probability that a single raindrop landing in the field hits the shed?
5. What is the probability that a raindrop does NOT hit the shed?	**6.** What is the probability that a raindrop misses both the shed and the tree? (Assume they do not overlap.)

Part B — Dartboard

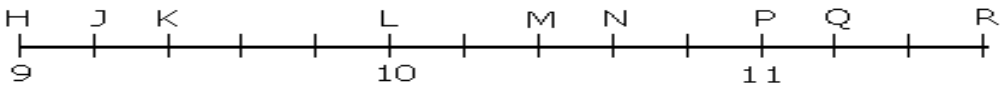
A dartboard has concentric circles (A, B, C, D) mounted on a 2 ft × 30 in. corkboard. Assume every dart lands on the corkboard. Round to the nearest tenth of a percent.



Circle	Radius	Ring Area
A	2 in.	
B	4 in.	
C	6 in.	
D	10 in.	

7. Find the area of circle A.	**8.** Find the area of ring B (circle B minus circle A).
9. Find the area of ring C (circle C minus A and B).	**10.** Find the area of ring D (rest of dartboard).
11. Find the area of the corkboard in square inches.	**12.** P(dart lands on any circle) =
13. P(dart lands on ring C or D) =	**14.** P(bull's-eye — circle A) =
15. P(dart lands only on ring C) =	

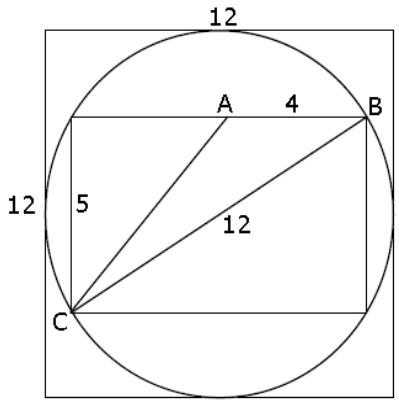
Part C — Number Line Probability



16. P(point on HR lies between J and P) =	17. P(point on HR lies between N and K) =
18. P(point on MN lies between J and K) =	19. P(point on LQ lies between M and P) =
20. A point on JR has a 25% probability of lying between _____ and _____.	21. A point on JQ has a 50% probability of lying between _____ and _____.

Part D — Inscribed Figures

Triangle ABC is inscribed in a rectangle, inscribed in a circle, inscribed in a square (side = 12). Find each probability. Round to the nearest tenth of a percent.



22. P(pebble lands only in triangle ABC)	23. P(pebble lands in rectangle but NOT triangle ABC)
24. P(pebble lands in circle but NOT rectangle)	25. P(pebble lands in square but NOT circle)

Lesson 3 — Compound Events: Independent & Dependent

Independent Events	Dependent Events
The outcome of one event does NOT affect the other. **P(A and B) = P(A) · P(B)** **Example:** Flip a coin AND roll a die. P(H and 4) = 1/2 · 1/6 = 1/12	The outcome of one event DOES affect the other (no replacement). **P(A then B) = P(A) · P(B | A)** **Example:** Draw 2 cards without replacement. P(ace then ace) = 4/52 · 3/51 = 1/221

Part A — Independent Events

1. Hal tosses a quarter three times. What is the probability of getting tails all three times?	2. A coin and a number cube are tossed. What is P(tails and a 4)?
3. A number cube is rolled twice. What is P(odd number then even number)?	4. A spinner has 8 equal sections numbered 1–8. It is spun twice. What is P(multiple of 3 both times)?
5. A bag has 5 red and 3 blue chips. A chip is drawn, replaced, then drawn again. What is P(red then blue)?	6. P(rain Saturday) = 0.3 and P(rain Sunday) = 0.5. Assuming independence, what is P(rain both days)?

Part B — Dependent Events (Without Replacement)

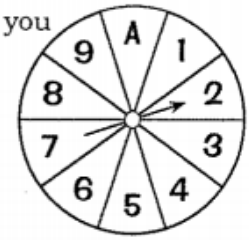
7. There are 3 apples and 5 oranges in a bag. What is P(2 apples in a row without replacement)?	8. From the same bag (3 apples, 5 oranges), what is P(orange then apple without replacement)?
9. There are 4 green marbles and 3 white marbles. What is P(2 white marbles without replacement)?	10. From the same bag (4 green, 3 white), what is P(green then white without replacement)?
11. From a deck of 52 cards, what is P(two aces drawn in a row without replacement)?	12. From 6 boys and 4 girls, 2 students are chosen at random without replacement. What is P(boy then girl)?

Which Italian Insects Often Fall in Love?

Find each correct answer in the set of answers under the exercise and cross out the letter above it.

1. Each time you spin this spinner, how many equally likely outcomes are there?

2. Find each probability if you spin the spinner once.



a. P(even number)

b. P(odd number)

c. P("A")

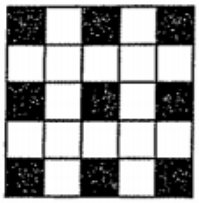
3. If you spin the spinner 100 times, about how many times would you expect it to stop on:

a. an even number **b.** an odd number

4. If you roll a regular 6-faced die 1200 times, about how many times would you expect to get a 4?

5. If a raindrop falls on this set of tiles, how many equally likely outcomes are there?

6. Find each probability if a raindrop falls on the tiles.



a. P(falling on black)

b. P(falling on white)

c. P(falling on green)

7. If 100 raindrops fall on the tiles, about how many of them would you expect to fall on:

a. a black tile **b.** a white tile

8. Jack rolled a regular 6-faced die three times and got 2 each time. What is the probability he will get 2 on the next roll?

K	I	T	R	A	L	O	N	E	O	M	D	E	S	R	A	S	N	T
200	$\frac{1}{2}$	64	$\frac{1}{3}$	$\frac{16}{25}$	50	$\frac{7}{10}$	$\frac{1}{6}$	10	$\frac{9}{25}$	60	$\frac{1}{10}$	0	25	$\frac{2}{5}$	24	36	$\frac{3}{8}$	40

9. Suppose a bag contains 12 green cubes, 5 blue cubes, and 3 yellow cubes. Find each probability if you choose one cube at random:

a. P(green) **b.** P(blue)

c. P(yellow) **d.** P(not blue)

10. If you spin this spinner 600 times, about how many times would you expect it to stop on:

a. $1

b. $5

c. $10

11. Jill tossed a coin 10 times and got heads every time. What is the probability she will gets heads on the next toss?

12. A traffic signal is green for 20 seconds, then amber for 5 seconds, then red for 30 seconds. When you reach the signal, what is the probability it is:

a. green **b.** amber

13. Suppose you do a survey to find the blood types of 200 people and obtain the results in the table. Based on this data, find the probability that a randomly chosen person has:

Blood Type	Number of People
O^+	76
O^-	14
A^+	68
A^-	12
B^+	18
B^-	4
AB^+	6
AB^-	2

a. Type O^+

b. Type A^-

c. Type B^-

d. Type AB^+ or AB^-

B	S	T	O	H	U	G	I	V	P	C	E	N	T	E	K	I	S	S
$\frac{3}{20}$	$\frac{19}{50}$	$\frac{7}{20}$	$\frac{1}{50}$	$\frac{3}{4}$	$\frac{1}{2}$	200	$\frac{3}{11}$	$\frac{3}{5}$	$\frac{1}{25}$	120	$\frac{1}{11}$	300	100	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{4}{11}$	$\frac{3}{50}$	$\frac{1}{3}$

Lesson 4 — Two-Way Tables & The Addition Rule

Key Formulas

P(A or B) = P(A) + P(B) − P(A and B) [overlapping events]
P(A or B) = P(A) + P(B) [mutually exclusive events]
P(B | A) = P(A and B) / P(A) [conditional probability: P of B given A]

Part A — Reading Two-Way Tables

The table shows grade distribution for student athletes at Jefferson High School.

Sport	A Average	B Average	Total
Field Hockey	15	4	19
Basketball	7	13	20
Football	2	22	24
Total	**24**	**39**	**63**

An athlete is randomly selected. Find each probability.

1. P(field hockey and B average)	**2. P(A average and football)**
3. P(basketball player)	**4. P(A average)**
5. P(field hockey OR A average)	**6. P(football | B average)**

The table below shows sports participation by grade at a high school (440 students total).

Grade	Track	Volleyball	Basketball	Tennis	No Sport	Total
9	12	18	15	9	66	120
10	6	20	12	2	95	135
11	15	11	8	5	61	100
12	7	6	10	12	50	85
Total	**40**	**55**	**45**	**28**	**272**	**440**

A student is randomly selected. Find each probability.

7. P(11th grade and tennis)	**8. P(9th grade)**
9. P(no sport and 12th grade)	**10. P(no sport OR 12th grade)**
11. P(plays volleyball | 10th grade)	**12. P(9th or 10th grade)**

Part B — Venn Diagram / OR Problems

Problem 13.
Of 100 students surveyed, 44 are male and 54 are in favor of a 4-day school week. Of those in favor, 20 are female. Let M = male, F = in favor of change.

a) P(male and NOT in favor)	b) P(male OR in favor)	c) P(female OR male)
= _____	= _____	= _____

Problem 14.
Of 100 students surveyed, 54 favor a 4-day school week and 27 plan to start their own business. Of those planning to start a business, 18 also favor the 4-day week. Let W = favor change, B = wants to start a business.

a) P(W and B)	b) P(W but NOT B)	c) P(neither W nor B)
= _____	= _____	= _____

Lessons 3 & 4 — Quiz Review

Find each probability.

1. Hal tosses a quarter three times. What is P(tails all three times)?

Answer: _____

2. Katie rolls a number cube twice. What is P(odd number then even number)?

Answer: _____

3. A bag has 3 apples and 5 oranges. What is P(2 apples without replacement)?

Answer: _____

4. Same bag — what is P(orange then apple without replacement)?

Answer: _____

A store tracks customers who enter a department and whether they make a purchase:

	Men	Women
Bought	7	4
Didn't Buy	5	9

5. What is P(a randomly chosen customer made a purchase)?

Answer: _____

6. What is P(a customer is a woman who did not buy)?

Answer: _____

Circle the best answer.

7. Coin and number cube tossed. P(tails and a 2, 3, or 4)?

A. 0.10 B. 0.25 C. 0.50 D. 0.75

8. Same toss. P(heads and an even number)?

A. 0.10 B. 0.25 C. 0.33 D. 0.50

Lesson 5 — Statistics: Center, Spread & Distributions

Measures of Center	Measures of Spread	Other Terms
Mean: sum ÷ count **Median:** middle value **Mode:** most frequent	**Range:** max − min **IQR:** Q3 − Q1 **Std Dev (σ):** avg distance from mean	**Outlier:** unusually far value **5-number summary:** min, Q1, med, Q3, max **Skew:** direction of the tail

Part A — Measures of Center and Spread

Number of green M&Ms per bag (8 groups): 3, 7, 5, 4, 6, 3, 8, 4

1. Arrange the data in order from least to greatest.

   _____

2. Mean = _____
3. Median = _____
4. Mode = _____
5. Range = _____
6. Q1 = _____
7. Q3 = _____
8. IQR = _____

9. Are there any outliers? Explain.

   _____

Part B — Standard Deviation

Standard deviation measures how spread out data values are from the mean.

Steps:
Step 1: Find the mean.
Step 2: Subtract the mean from each value to get each deviation.
Step 3: Square each deviation.
Step 4: Find the mean of the squared deviations (divide by n − 1 for sample data).
Step 5: Take the square root — this is the standard deviation.

10. Use the green M&M data to complete the standard deviation table.

Value (x)	x – mean	$(x - mean)^2$	Notes
3			
7			
5			
4			
6			
3			
8			
4			
Sum:			**σ = _____**

Part C — Frequency Distributions & Normal Distribution

11. Using your M&M data, create a frequency distribution bar graph for your assigned color. Label both axes.

Frequency Table

Quantity	Frequency

Bar Graph

Freq.

Quantity per Package

12. Does your frequency distribution resemble a normal distribution? Explain.

_____

_____

The Normal Distribution — Empirical Rule (68-95-99.7)

In a normal (bell-shaped) distribution:

68% of data falls within 1 standard deviation of the mean

95% of data falls within 2 standard deviations of the mean

99.7% of data falls within 3 standard deviations of the mean

13. A class of 200 students took a test. Scores were normally distributed: mean = 75, standard deviation = 8.

a) What scores fall within 1 standard deviation of the mean? _____	b) What scores fall within 2 standard deviations? _____
c) About how many students scored between 67 and 83? _____	d) About how many students scored above 91? _____

14. Heights of adult males are normally distributed with mean 70 in. and standard deviation 3 in. About what percent of males are between 64 and 76 inches tall? Show your reasoning.

_____

_____