Given: $f(x) = \cos(x)$

a. Determine a **line** $P(x)$ that closely matches the characteristics of $f(x)$ at the point $(0, 1)$.

$P(x) =$ ______________________

b. Use Geogebra to visualize how $f(x)$ and $P(x)$ interact.

c. Fill in the following table (rounding to 3 decimal places when necessary).

x	$f(x)$	$P(x)$	$\lvert f(x) - P(x)\rvert$
0			
$.1$			
$.5$			
1			

Given: $f(x) = \cos(x)$

a. Determine a **parabola** $P(x)$ that closely matches the characteristics of $f(x)$ at the point $(0, 1)$.

$P(x) =$ ______________________

b. Use Geogebra to visualize how $f(x)$ and $P(x)$ interact.

c. Fill in the following table (rounding to 3 decimal places when necessary).

x	$f(x)$	$P(x)$	$\lvert f(x) - P(x)\rvert$
0			
$.1$			
$.5$			
1			

Given: $g(x) = e^x$

a. Determine a **line** $P(x)$ that closely matches the characteristics of $f(x)$ at the point $(0, 1)$.

$P(x) =$ ____________________

b. Use Geogebra to visualize how $f(x)$ and $P(x)$ interact.

c. Fill in the following table (rounding to 3 decimal places when necessary).

x	$g(x)$	$P(x)$	$\lvert g(x) - P(x)\rvert$
0			
$.1$			
$.5$			
1			

Given: $g(x) = e^x$

a. Determine a **parabola** $P(x)$ that closely matches the characteristics of $f(x)$ at the point $(0, 1)$.

$P(x) =$ ____________________

b. Use Geogebra to visualize how $f(x)$ and $P(x)$ interact.

c. Fill in the following table (rounding to 3 decimal places when necessary).

x	$g(x)$	$P(x)$	$\lvert g(x) - P(x)\rvert$
0			
$.1$			
$.5$			
1			

Given: $f(x) = \cos(x)$

Determine a **fourth degree polynomial**, $P(x)$, that closely matches the characteristics of $f(x)$ when centered at $x = 0$.

x	$f(x)$	$P(x)$	$\lvert f(x) - P(x) \rvert$
0			
$.1$			
$.5$			
1			

Given: $g(x) = e^x$

Determine a **fourth degree polynomial**, $P(x)$, that closely matches the characteristics of $f(x)$ when centered at $x = 0$.

x	$g(x)$	$P(x)$	$\lvert g(x) - P(x) \rvert$
0			
$.1$			
$.5$			
1			

Taylor Series as an infinite sum.

Write a Taylor/**Maclaurin (centered at $\boldsymbol{x = 0}$)** Series polynomial function with at least 4 terms for each of the following functions. Then write the polynomial function as an infinite series. Use a separate sheet of paper for work.

1. $f(x) = \cos(x)$		
2. $f(x) = e^x$		
3. $f(x) = \ln(1+x)$		
4. $f(x) = \sin(x)$		
5. $f(x) = \frac{1}{1-x}$		
6. $f(x) = tan^{-1}x$		

Investigating the Taylor/MacLaurin Series for $f(x) = tan^{-1}x$ centered at $x = 0$.

$f(x) = \dfrac{1}{1-x} = \sum\limits_{n=0}^{\infty}$	$g(x) = tan^{-1}x = \sum\limits_{n=0}^{\infty}$
$f'(x) =$	$g'(x) = \dfrac{1}{1+x^2} = \dfrac{1}{1-(\quad)} = \sum\limits_{n=0}^{\infty}$

Power Series and Radius of Convergence

A Power Series centered at $x = a$ is a function of x and can be written as:

$$f(x) = C_0 + C_1(x-a)^1 + C_2(x-a)^2 + C_3(x-a)^3 + \cdots = \sum_{n=0}^{\infty} C_n(x-a)^n$$

* For each Power Series, a. Write out the first 4 terms of the series, then b. Use the Ratio Test to determine both the Interval of Convergence and Radius of Convergence.

1. $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

2. $\sum_{n=1}^{\infty} \frac{n^3(x+5)^n}{6^n}$

3. $\sum_{n=0}^{\infty} \frac{(-1)^n(n)}{4^n}(x+3)^n$

4. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

5. $\sum_{n=1}^{\infty} \frac{2^n}{n}(4x-8)^n$

6. $\sum_{n=0}^{\infty} \frac{x^{2n}}{(-3)^n}$

7. $\sum_{n=0}^{\infty} n!\,(x-2)^n$

8. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \cdot (x-1)^n$

9. $\sum_{n=1}^{\infty} \frac{(x-6)^n}{n^n}$

10. $\sum_{n=0}^{\infty} 2^{2n} \cdot x^{2n}$