Sequences and Series

Write each decimal as a fraction in simplest form.

1. 0.4 $\frac{4}{10}$	2. $0.\overline{3}$ $\frac{1}{3}$	3. 0.333 $\frac{333}{1000}$
4. $0.15\overline{6}$ $100x = 15.\overline{6}$ $1000x = 156.\overline{6}$ 15 $900x = 141$ $x = \frac{141}{900} = \frac{47}{300}$	5. $0.12\overline{34}$ $100x = 12.\overline{34}$ $10000x = 1234.\overline{34}$ $9900x = 1222$ $x = \frac{1222}{9900}$	6. $0.\overline{9}$ 1

For each arithmetic or geometric sequence, determine the next three values and then write both an explicit and a recursive formula.

7. 42, 747, 1452, 2157, … 2862, 3567, 4272 $t(n) = -663 + 705n$ $t(1) = 42$ $t(n+1) = t(n) + 705$	8. 42, 294, 2058, 14406, … 100842, 705894, 4941258 $t(n) = 6 \cdot 7^n$ $t(1) = 42$ $t(n+1) = t(n) \cdot 7$
9. 40, 20, 10, 5, … $\frac{5}{2}$, $\frac{5}{4}$, $\frac{5}{8}$ $t(n) = 80\left(\frac{1}{2}\right)^n$ $t(1) = 40$ $t(n+1) = t(n) \cdot \frac{1}{2}$	10. 753, 726, 699, 672, … 645, 618, 591 $t(n) = 780 - 27n$ $t(1) = 753$ $t(n+1) = t(n) - 27$

Determine the arithmetic and geometric means for each. Arithmetic: 9, <u>12</u>, <u>15</u>, 18 $9 + 3x = 18$ $3x = 9$ $x = 3$ Geometric: 9, <u>$9\sqrt[3]{2}$</u>, <u>$9\sqrt[3]{4}$</u>, 18 $9 \cdot x^3 = 18$ $x^3 = 2$	Determine the arithmetic and geometric means for each. Arithmetic: 2, <u>6</u>, <u>10</u>, <u>14</u>, 18 $2 + 4x = 18$ $4x = 16$ $x = 4$ Geometric: 2, <u>$2\sqrt{3}$</u>, <u>6</u>, <u>$6\sqrt{3}$</u>, 18 $2 \cdot x^4 = 18$ $x^4 = 9$ $x = \sqrt{3}$

Notes:

11. Write an explicit and recursive formula for the Fibonacci Sequence 1, 1, 2, 3, 5, 8, 13, 21, ...

RECURSIVE

$t(1) = 1$

$t(2) = 1$

$t(n+2) = t(n) + t(n+1)$

12. Find the sum of the first one hundred positive integers.

5050

13. Find the sum of the first thirty multiples of two.

930

14. Find the sum of the first thirty powers of two.

$2^{31} - 2$

15. Find the sum of the first 27 terms in the sequence.
42, 747, 1452, 2157, ...

$42 + 747 + 1452 + \ldots + 18372$

248589

16. Find the sum of the first 27 terms in the sequence.
40, 20, 10, 5, ...

$80\left(\frac{1}{2}\right)^{27}$

$S = 40 + 20 + 10 + 5 + \ldots + 80\left(\frac{1}{2}\right)^{27}$

$\frac{1}{2}S = 20 + 10 + \ldots + 80\left(\frac{1}{2}\right)^{28}$

$\frac{1}{2}S = 40 - 80\left(\frac{1}{2}\right)^{28}$

$S = 2\left[40 - 80\left(\frac{1}{2}\right)^{28}\right]$

17. Evaluate

$$\sum_{k=1}^{7} 4(3)^k$$

$S = 12 + 36 + \ldots + 4 \cdot 3^7$

$3S = 36 + \ldots + 4 \cdot 3^8$

$2S = 4 \cdot 3^8 - 12$

$S = \frac{4 \cdot 3^8 - 12}{2}$

More Practice...For each arithmetic or geometric sequence, determine the next three values and then write both an explicit and a recursive formula.

18. $30, 60, 120, 240, \ldots$ 480, 960, 1920

$t(n) = 15(2)^n$

$t(1) = 30$

$t(n+1) = t(n) \cdot 2$

19. $100, 105, 110, 115, \ldots,$ 120, 125, 130

$t(n) = 95 + 5n$

$t(1) = 100$

$t(n+1) = t(n) + 5$

20. $4, \frac{-16}{3}, \frac{64}{9}, \frac{-256}{27}, \ldots$ $\frac{1024}{81}$, $\frac{-4096}{243}$, $\frac{16384}{729}$

$t(n) = -3\left(\frac{-4}{3}\right)^n$

$t(1) = 4$

$t(n+1) = t(n) \cdot \frac{-4}{3}$

21. $-6, -4+\pi, -2+2\pi, 3\pi, \ldots,$ $2+4\pi$, $4+5\pi$, $6+6\pi$

$t(n) = -8 - \pi + (2+\pi)n$

$t(1) = -6$

$t(n+1) = t(n) \cdot + 2 + \pi$

The Golden Ratio

Draw a line segment split into two parts (labelled a and b where $a > b$) that closely resembles the following...**The whole length divided by the long part is also equal to the long part divided by the short part.** The Golden Ratio, φ, Phi is the value of these ratios. (This segment does not need to be perfect. It is just to guide you through the rest of this activity.)

Write a proportion (in terms of a and b) representing the golden ratio statement.

$$\frac{a+b}{a} = \frac{a}{b}$$

Using your proportion, set $b = 1$ and solve for a.

$$\frac{a+1}{a} = \frac{a}{1}$$

$$a^2 = a + 1$$

$$a^2 - a - 1 = 0$$

$$a = \frac{1 \pm \sqrt{1+4}}{2}$$

$$a = \frac{1+\sqrt{5}}{2}$$

a is the Golden Ratio! Do you see why a is the value of the golden ratio?

Test these out:

$$2 \cdot \sin(54°)$$

$$1 + \frac{1}{\varphi}$$

Test these out:

$$1 + \frac{1}{1+\frac{1}{\varphi}}$$

$$1 + \frac{1}{1+\frac{1}{1+\frac{1}{\varphi}}}$$

Examine this function for positive integers of x.

$$y = \left\lfloor \frac{\varphi^x}{\sqrt{5}} + \frac{1}{2} \right\rfloor$$

Greatest Integer Function
(aka Floor Function)
MATH->NUM->5

Neat Link For The Golden Ratio in Nature, Art, and Design: https://www.canva.com/learn/what-is-the-golden-ratio/