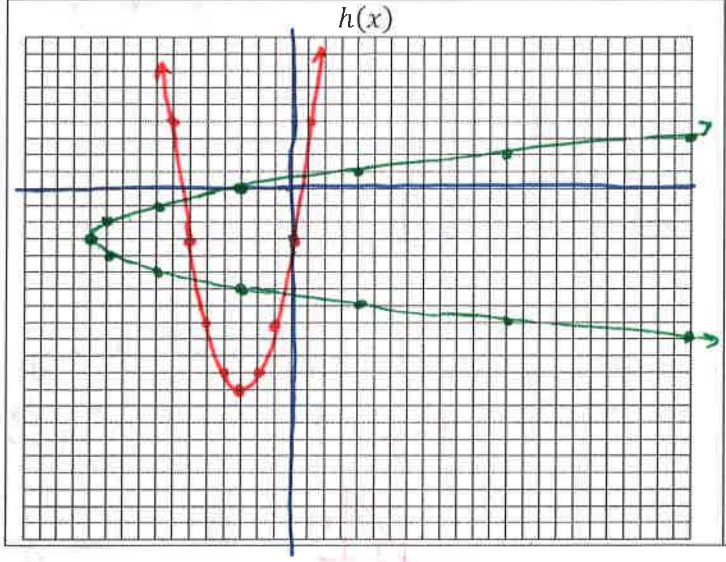
1. What are inverse relations/functions?	2. How do the graphs of inverse relations/functions compare?

$(-3, -12$

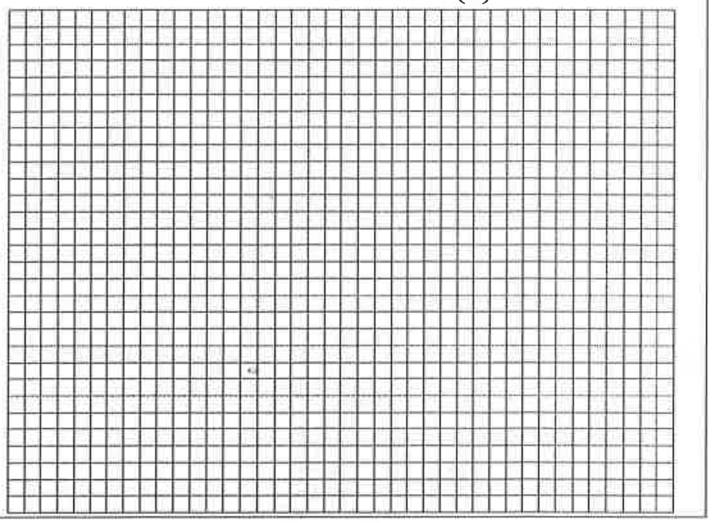
$-\frac{6}{2}$

3. Graph the function $h(x) = x^2 + 6x - 3$ and its inverse by using tables.

$h(x)$



The inverse of $h(x)$



* Find the inverse of each function.

4. $f(x) = 3x - 7$	5. $g(x) = x^3 - 7$	6. $h(x) = x^2 + 6x - 3$
$x = 3y - 7$ $\frac{x+7}{3} = y$	$x = y^3 - 7$ $\sqrt[3]{x+7} = y$	$x = y^2 + 6y + 9 - 12$ $x = (y+3)^2 - 12$ $\pm\sqrt{x+12} - 3 = y$

*Use the results from above to assist you on each of the following.

7. If f$(x) = 3x - 7$, find $(f^{-1})'(5)$.	8. If $g(x) = x^3 - 7$, find $(g^{-1})'(20)$.	9. If $h(x) = x^2 + 6x - 3$, find $(h^{-1})'(13)$.
$f^{-1}(x) = \frac{x+7}{3}$ $(f^{-1})'(x) = \frac{1}{3}$ $\frac{1}{3}$	$(g^{-1})'(x) = \frac{1}{3}(x+7)^{-2/3}$ $(g^{-1})'(20) = \frac{1}{3\sqrt[3]{27}^2}$ $= \boxed{\frac{1}{27}}$	$h'(x) = 2x + 6$ $h'(2) = 10$ $(h^{-1})'(13) = \frac{1}{10}$

Begin to think about some possible shortcuts for this process. Any thoughts?

continued

$\begin{array}{r} {}^{5}16 \\ \times 9 \\ \hline 144 \end{array}$ $3\overline{)192}$ 64 $x = 4$

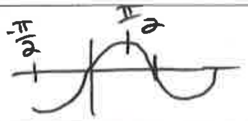
$0 = 2x^3 + 7x - 9 \longrightarrow x = 1$

$(1,5)$ on g corresponds w/ $(5,1)$ on g^{-1}

10. If $p(x) = 3x^3 + 5$, find $(p^{-1})'(197)$.	11. If $q(x) = 2x^3 + 7x - 4$, find $(q^{-1})'(5)$.
$p'(x) = 9x^2$ $p'(4) = 144$ $(p^{-1})'(197) = \frac{1}{144}$	$g'(x) = 6x^2 + 7$ $g'(1) = 13$ $(g^{-1})'(5) = \frac{1}{13}$

$x = 2y^3 + 7y - 4$

1



Inverse Trigonometric Functions and Their Inverses

12.

$f(x) = \sin x$	$f^{-1}(x) = sin^{-1}x$
Domain: $\mathbb{R}$ Range: $[-1, 1]$	Domain: $[-1, 1]$ Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

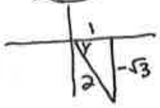
13. If $f(x) = \sin x$, then $(f^{-1})'\left(\frac{-\sqrt{3}}{2}\right) =$

$y = \sin x \longrightarrow x = \sin y$

$1 = \cos y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$\boxed{2}$



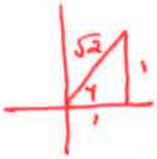
14. If $g(x) = sin^{-1}(x)$, then $g'\left(\frac{-\sqrt{3}}{2}\right) =$

$\leftarrow$ same question $\boxed{2}$

15. If $h(x) = tan^{-1}x$, then $h'(1) =$

$y = \tan x \longrightarrow x = \tan y$

$\tan y = \frac{1}{1}$



$1 = \sec^2 y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$= \cos^2 y$

$h'(1) = \left(\frac{1}{\sqrt{2}}\right)^2$

$h'(1) = \boxed{\frac{1}{2}}$

Assume 1st quadrant for the next set of questions.

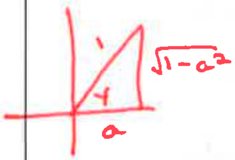
16. If $p(x) = cos^{-1}x$, then $p'(a) =$

$y = \cos x \longrightarrow x = \cos y$

$\cos y = \frac{a}{1}$

$1 = -\sin y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-1}{\sin y}$



$p'(a) = \frac{-1}{\sqrt{1-a^2}}$

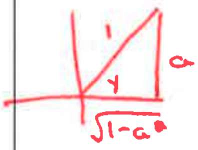
$= \boxed{\frac{-1}{\sqrt{1-a^2}}}$

17. If $r(x) = sin^{-1}x$, then $r'(a) =$

$y = \sin x \longrightarrow x = \sin y$

$\sin y = \frac{a}{1}$

$\frac{dy}{dx} = \frac{1}{\cos y}$



$r'(a) = \frac{1}{\sqrt{1-a^2}}$

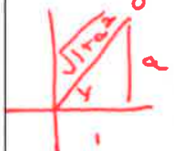
$= \boxed{\frac{1}{\sqrt{1-a^2}}}$

18. If $l(x) = tan^{-1}x$, then $l'(a) =$

$y = \tan x \longrightarrow x = \tan y$

$\tan y = \frac{a}{1}$

$1 = \sec^2 y \frac{dy}{dx}$



$\frac{dy}{dx} = \cos^2 y$

$l'(a) = \left(\frac{1}{\sqrt{1+a^2}}\right)^2$

$= \boxed{\frac{1}{\sqrt{1+a^2}}}$

Intro to Parametric Equations

1. Given: $x = t$ and $y = t^2$
Write the equation for the tangent line when $t = -2$.
Write the Cartesian Equation for this set of parametric equations.

$dx = dt$ $\qquad \frac{dy}{dt} = 2t$

$\frac{dt}{dx} = 1$

$t = -2$ $\quad dy/dt$ $\quad x = -2$ $\qquad y = 4$

or dx/dt

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 2t \cdot 1$$

$$\frac{dy}{dx}\Big|_{t=-2} = -4$$

$$y - 4 = -4(x+2)$$

$$y = x^2$$

2. Given: $x = \cos t$ and $y = \sin t$
Write the equation for the tangent line when $t = \frac{\pi}{3}$.
Write the Cartesian Equation for this set of parametric equations.

$\frac{dx}{dt} = -\sin t$ $\qquad \frac{dy}{dt} = \cos t$

$t = \frac{\pi}{3}$ $\qquad x = \cos\frac{\pi}{3} = \frac{1}{2}$ $\qquad y = \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t}$$

$$\frac{dy}{dx}\Big|_{t=\frac{\pi}{3}} = \frac{\cos(\frac{\pi}{3})}{-\sin(\frac{\pi}{3})} = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$y - \frac{\sqrt{3}}{2} = \frac{-1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$$

$$y = \sin(\cos^{-1}(x))$$

$$y = \sqrt{1-x^2}$$

3. Given: $x = 4t - 3$ and $y = 16t^2 - 9$ for $-\infty < t < \infty$

Write the equation for the tangent line when $t = \frac{1}{2}$.

Write the Cartesian Equation for this set of parametric equations.

$\frac{dy}{dt} = 32t$ $\frac{dx}{dt} = 4$

$x = 4(\frac{1}{2}) - 3 = -1$

$y = 16(\frac{1}{2})^2 - 9 = -5$

$\frac{dy}{dx} = \frac{32t}{4} = 8t$

$\frac{dy}{dx}\Big|_{t=\frac{1}{2}} = 4$

$\boxed{y + 5 = 4(x+1)}$

$\boxed{y = 16\left(\frac{x+3}{4}\right)^2 - 9}$

$\frac{1}{3/4} - 1 =$

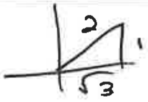
4. Given: $x = sec^2 t - 1$ and $y = \tan t$ for $-\frac{\pi}{2} < t < \frac{\pi}{2}$

Write the equation for the tangent line when $t = \frac{\pi}{6}$.

Write the Cartesian Equation for this set of parametric equations.

$x(\frac{\pi}{6}) \frac{1}{(\cos(\frac{\pi}{6}))^2} - 1 = + \frac{1}{3}$

$y(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$



$\frac{dy}{dt} = sec^2 t = \frac{1}{[\cos t]^2}$

$\frac{dx}{dt} = 2\sec(t) \cdot \sec(t) \cdot \tan(t)$

$\frac{dy}{dt}\Big|_{t=\frac{\pi}{6}} = \frac{1}{(\frac{\sqrt{3}}{2})^2} = \frac{4}{3}$

$= \frac{2}{[\cos t]^2} \frac{\sin(t)}{\cos(t)}$

$\frac{dx}{dt}\Big|_{t=\frac{\pi}{6}} = \frac{2(\frac{1}{2})}{(\sqrt{3}/2)^3} = \frac{1}{\frac{3\sqrt{3}}{8}} = \frac{8}{3\sqrt{3}}$

$\frac{4}{3} \cdot \frac{3\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$

~~[illegible]~~

~~[illegible]~~

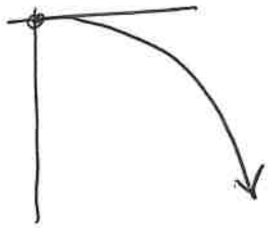
$\boxed{y - \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}(x + \frac{1}{3})}$

$\boxed{y = \sqrt{x}}$

5. The motion of a particle travelling in two-dimensional space can be described by the following parametric equations where horizontal and vertical position are measured in meters and time is measured in seconds.
$x = t^2$ and $y = -t^3$ for $t \geq 0$

a. Determine a Cartesian Equation for this particle in two-dimensional space.
b. Use graph paper or a graphing utility to visualize the motion of this particle.
c. Determine an equation for a the tangent line of this curve when $t = 4$.
d. Determine the speed of this particle at times $t = 1$ and $t = 4$.

b.



a. $\sqrt{x} = t \qquad y = -(\sqrt{x})^3 = -x^{3/2}$

c. $\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = -3t^2$

$\frac{dy}{dx} = \frac{-3t^2}{2t} = \frac{-3t}{2}$

$\boxed{y + 64 = -6(x - 16)}$

$\frac{dy}{dx}\Big|_{t=4} = -6$

d. $v(t) = < 2t, -3t^2 >$

SPEED $(4) = \sqrt{8^2 + [-48]^2} \approx \boxed{48.662 \text{ m/s}}$

SPEED AT $\Big|_{t=1} = \sqrt{2^2 + (3)^2}$

$\approx \boxed{\sqrt{13} \text{ /s}}$

6. Given the Cartesian Equation $y = x^2 + 7$.
a. Determine at least three differing sets of parametric equations that could describe the given Cartesian Equation.
b. Determine an equation for the tangent line at $x = -2$ using at least two different methods.

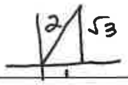
$\boxed{x = t \qquad y = t^2 + 7}$

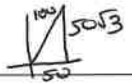
CHEATING ☺

$\boxed{x = \sqrt{t} \qquad y = t + 7}$

$\boxed{x = \sqrt{\sqrt{t} - 7} \qquad y = \sqrt{t}}$

$\boxed{x = \sqrt{t - 3} \qquad y = t + 4}$





7. A person on top of a 650 foot cliff throws an object up at an angle of 60 degrees from the horizontal with an initial velocity of of 100 feet per second. Given: Air resistance is neglected, acceleration of gravity (vertical) is -32 feet/second per second.

Horizontal Components

a. Determine the horizontal component for the initial velocity. 50 ft/s

b. Write acceleration, velocity, and position equations $a_x(t), v_x(t), s_x(t)$ respectively.

$a_x(t) = 0$

$v_x(t) = 50$

$s_x(t) : 50t + 0$

Vertical Components

a. Determine the vertical component for the initial velocity. $50\sqrt{3}$ ft/s

b. Write acceleration, velocity, and position equations $a_y(t), v_y(t), s_y(t)$ respectively.

$a_y(t) = -32$

$v_y(t) = -32t + 50\sqrt{3}$

$s_y(t) = -16t^2 + 50\sqrt{3}\,t + \overset{650}{\cancel{100}}$

Application Questions

8.
a. Write an equation for the vertical position of the object with respect to the horizontal distance.
b. What is the objects vertical position when its horizontal position is 350 feet?
c. What is the objects horizontal position when its vertical position is 683 feet?
d. At what time does the object reach its highest point?

e. How high is the object at the peak of its path?
f. How long did it take the object to hit the ground?
g. How fast is the object travelling when its horizontal position is 350 feet?
h. How fast is the object travelling when its vertical position is 683 feet?
i. How fast is the object travelling the instant before it hits the ground?

a. $x = 50t \quad t = \frac{x}{50}$

$y = -16\left(\frac{x}{50}\right)^2 + 50\sqrt{3}\left(\frac{x}{50}\right) + 650$

b. $y(350) \approx$ 472.218 ft

c. when $y(t) = 683$, $x \approx 20, 624,$ 250 ft

d. $t \approx 2.706$ seconds

e. 767.188 ft

f. $t \approx 9.631$ seconds

g. $\vec{v}(7) = \langle 50, -137.397\rangle$

speed ≈ 146.212 ft/s

h. $\vec{v}(5) = \langle 50, -73.397\rangle$

speed $= 88.809$ ft/s

i. $\vec{v}(9.631) = \langle 50, -221.589\rangle$

speed $= 227.160$ ft/s

The Second Derivative of a Parametric Equation

9. Given: $x = t$ and $y = t^2$

a. Write the Cartesian Equation for this set of parametric equations.

$y = x^2$

$\frac{dx}{dt} = 1$ $\quad \frac{dy}{dt} = 2t$

$\frac{d}{dt}\left(\frac{dy}{dt}\right) = 2$

b. Use the Cartesian Equation to determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

$\frac{dy}{dx} = 2x$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = 2$

c. Use the parametric equations to determine both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. (Example: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$)

$\frac{d}{dx}\left(\bigcirc\!\!\!\!\!\; y\right) = \frac{\frac{d}{dt}(\bigcirc\!\!\!\!\!\; y)}{dx/dt}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

d. What do you notice when comparing parts b. and c.

$\frac{dy}{dt} = 2t$ $\quad \frac{dx}{dt} = 1$

$\frac{dy}{dx} = \frac{2t}{1} = 2t$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(2t)}{dx/dt}$

Notes:

$= \bigcirc\!\!\!\!\!\!\; \frac{2}{1}$

Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

10. Given: $x = \cos t$ and $y = \sin t$

$\frac{dx}{dt} = -\sin t$

$\frac{dy}{dt} = \cos t$

$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left[-\cot(t)\right] = \csc^2(t)$

$\frac{dy}{dx} = \frac{\cos(t)}{-\sin(t)} = \boxed{-\cot(t)}$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\csc^2(t)}{-\sin t} = \boxed{\frac{-1}{\sin^3 t}}$

11. Given: $x = 4t - 3$ and $y = 16t^2 - 9$ for $-\infty < t < \infty$

$\frac{dy}{dt} = 32t$

$\frac{dx}{dt} = 4$

$\frac{d}{dt}\left(\frac{dy}{dx}\right) = 8$

$\frac{dy}{dx} = \frac{32t}{4} = 8t$

$\frac{d^2y}{dx^2} = \frac{8}{4} = 2$

12. Given: $x = sec^2 t - 1$ and $y = \tan t$ for $-\frac{\pi}{2} < t < \frac{\pi}{2}$

$\frac{dy}{dt} = \sec^2(t)$

$\frac{dx}{dt} = 2\sec(t) \cdot \sec(t)\tan(t)$

$\frac{dy}{dx} = \frac{\sec^2(t)}{2\sec^2(t) \cdot \tan(t)} = \frac{1}{2}\cot(t)$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{-\frac{1}{2}\csc^2(t)}{2\sec^2(t)\tan(t)} = \boxed{-\cot^3(t)}$