

MODULE 9 – EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Module 9: Lessons 1 & 2 – Add/Subtract/Multiply Divide Rational Expressions

Add/Subtract Fractions

Evaluate.

$$1. \frac{1}{2} + \frac{2}{5}$$

$$2. \frac{3}{7} - \frac{1}{4}$$

$$\begin{array}{r} 3\frac{4}{4} \\ \frac{6}{5} + \frac{7}{2} - \frac{3}{4} \\ \hline 5\frac{5}{5} \end{array}$$

4. $\frac{1}{4} + \frac{2}{x}$

$$\frac{24}{20} + \frac{70}{20} - \frac{15}{20}$$

$$\frac{79}{20}$$

$$\frac{x+8}{4x}$$

Multiply/Divide Fractions

Evaluate each.

$$5. \frac{3}{8} \cdot \frac{2}{7}$$

6. $\frac{15}{4} \cdot \frac{14}{9}$

$$7. \frac{x^2}{3} \cdot \frac{21}{x}$$

$$\frac{3}{28}$$

$$\frac{35}{6}$$

7x

8.	$\frac{5}{2} \div \frac{1}{2}$
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9. $\frac{10}{3} \div \frac{14}{21}$

10. $\frac{x^3}{15} \div \frac{x}{5}$

5

$$\frac{10}{3} \cdot \frac{21}{14}$$

5

$$\frac{x^2}{3}$$

Multiply/Divide Rational Expressions

*Simplify each Rational Expression. Discuss the idea of quantities vs. terms.

11. $\frac{x+7}{x} \cdot \frac{x-7}{(x-7)(x+7)}$

12. $\frac{(x+6)(x-2)}{(x+1)} \div \frac{(x-7)(x-2)}{(x-7)}$

13. $\frac{x+8}{x^2-6x-27} \cdot \frac{x^2+8x+15}{2x+16}$

$$\frac{x+8}{(x-9)(x+3)} \cdot \frac{(x+3)(x+5)}{2(x+8)}$$

$$\frac{1}{x}$$

$$\frac{x+6}{x+1}$$

$$\frac{x+5}{2(x-9)}$$

Add/Subtract Fractions

*Simplify each Rational Expression. Identify restrictions for x .

14. $\frac{3}{x+1} + \frac{7}{x-2}$

15. $\frac{x}{x+1} - \frac{7}{x-2}$

$$\frac{(x-2)x}{(x-2)x+1} - \frac{7}{x-2} \frac{(x+1)}{(x+1)}$$

$$\frac{x(x-2)}{(x-2)(x+1)} - \frac{7(x+1)}{(x-2)(x+1)}$$

$$\frac{10x+1}{(x-2)(x+1)}$$

$$\frac{x^2 - 2x - (7x + 7)}{(x-2)(x+1)} = \frac{x^2 - 9x - 7}{(x-2)(x+1)}$$

Working with Opposite Expressions.

*Simplify each Rational Expression.

16. $\frac{1}{(x-7)} \cdot \frac{(7-x)}{8}$ $\frac{1}{(x-7)} \cdot \frac{-1(x-7)}{8}$ $\frac{-1}{8}$	17. $\frac{18x^6}{x-5} \div \frac{12x}{5-x}$ $\frac{18x^6}{(x-5)} \cdot \frac{-1(x-5)}{12x}$ $\frac{-3x^5}{2}$
18. $\frac{5}{2x-9} + \frac{3x}{9-2x}$ $\frac{5}{2x-9} + \frac{3x}{-1(2x-9)}$ $\frac{5}{2x-9} + \frac{-3x}{2x-9}$ $\frac{5-3x}{2x-9}$	19. $\frac{c+3}{3c-2} + \frac{2c+1}{2-3c} + \frac{3c-1}{-2+3c}$ $\frac{c+3}{3c-2} + \frac{2c+1}{-1(3c-2)} + \frac{3c-1}{3c-2}$ $\frac{c+3 - 2c-1 + 3c-1}{3c-2} = \frac{2c+1}{3c-2}$

Working with other Occurrences with Rational Expressions.

*Simplify each Rational Expression. (Talk about how to check answers with a calculator.) Identify restrictions for x.

20. $\frac{4}{3} + \frac{x}{12}$ $\frac{4}{12} + \frac{x}{12}$ $\frac{4+x}{12}$	21. $\frac{3}{x+2} + 2 \frac{(x+2)}{(x+2)}$ $\frac{3}{x+2} + \frac{2(x+2)}{x+2}$ $\frac{3+2x+4}{x+2} = \frac{2x+7}{x+2}$	22. $\frac{3x}{x^2+3x-10} - \frac{6}{x^2+3x-10}$ $\frac{3x-6}{x^2+3x-10}$ $\frac{3(x-2)}{(x+5)(x-2)} = \frac{3}{x+5}$
23. $\frac{x-1}{x^2-1} - \frac{2x}{x^2-2x+1}$ $\frac{x-1}{(x+1)(x-1)} - \frac{2x}{(x-1)(x-1)}$ $\frac{(x-1)}{(x+1)(x-1)} - \frac{2x}{(x-1)(x-1)} \frac{(x+1)}{(x+1)}$ $\frac{x^2 - 2x + 1 - 2x(x+1)}{(x-1)^2(x+1)}$ $= \frac{-x^2 - 4x + 1}{(x-1)^2(x+1)}$	24. $\frac{\frac{x^2+12x+35}{x^2-3x-40}}{\frac{x^2-3x-40}{x^2+9x+14}} \div \frac{x^2+9x+14}{x^2-64}$ $\frac{x^2+12x+35}{x^2-3x-40} \div \frac{x^2+9x+14}{x^2-64}$ $\frac{x^2+12x+35}{x^2-3x-40} \cdot \frac{x^2-64}{x^2+9x+14}$ $\frac{(x+5)(x+7)}{(x-8)(x+5)} \cdot \frac{(x+8)(x-8)}{(x+2)(x+7)}$ $\frac{x+8}{x+2}$	

Intro: Cross-Multiply each of the following to solve for x .* Solve for x .

25. $\frac{x}{9} = \frac{5}{3}$	26. $\frac{7}{4} = \frac{1}{x}$	27. $5 = \frac{3x+1}{2}$
$x = 15$	$x = \frac{4}{7}$	$x = 3$

Reattempt each problem by multiplying both sides of equation by denominator values until denominators are gone.

28. $\frac{7}{4} = \frac{1}{x}$	29. $5 = \frac{3x+1}{2}$
$x = \frac{4}{7}$	$x = 3$

Cross-Multiplication is ...

30. Solve for x .

$$\frac{x}{3} + \frac{2x}{5} = 10$$

a. Combine fractions on each side of equation and then cross-multiply.

$$\frac{5}{5} \frac{x}{3} + \frac{2}{3} \frac{2x}{5} = 10$$

$$\frac{5x}{15} + \frac{6x}{15} = 10$$

$$\frac{11x}{15} = \frac{10}{1}$$

$$11x = 150$$

$$x = \frac{150}{11}$$

b. Multiply both sides of the equation by the denominator values.

$$3 \left(\frac{x}{3} + \frac{2x}{5} \right) = (10) 3$$

$$5 \left(x + \frac{6x}{5} \right) = (30) 5$$

$$5x + 6x = 150$$

$$11x = 150$$

$$x = \frac{150}{11}$$

Focus: Multiply both sides of the equation by the denominator values.

Identify any *solution restrictions* before beginning.

Solve for x.

31. $\frac{a}{7} - \frac{a-5}{2} = 3$

$a = -\frac{7}{5}$

32. $\frac{2}{3c} = \frac{5+c}{4c} - \frac{5}{12}$

$c = \frac{7}{2}$

33. $\frac{2}{x+3} + \frac{3}{x+4} = \frac{7}{x^2+7x+12}$

$x = -2$

34. $\frac{x-1}{x+1} - \frac{6}{x-3} = 3$

$x = -3, 1$

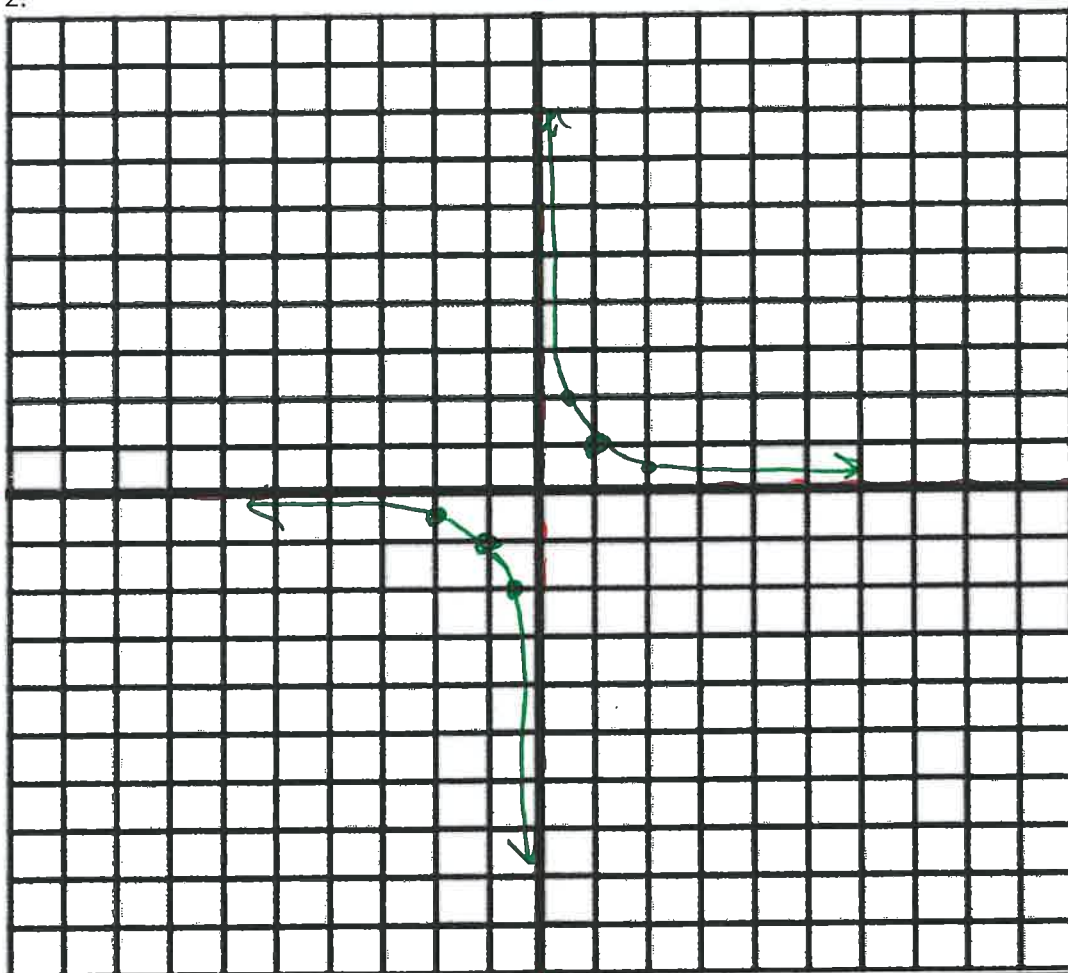
Module 9: Lessons 3 & 4 – Graphing Reciprocal and Rational Functions

Investigating the function $f(x) = \frac{1}{x}$

1. Use your calculator to graph the function $f(x) = \frac{1}{x}$

- Use Zoom 6 to see what the function looks like.
- Use Table to accurately sketch the function in the cell below.
- What are the coordinates of the two easiest points to remember from this table?

2.



3. Remember the term asymptote from the unit on exponents and logarithms. Remember that an asymptote is an imaginary line that a function approaches but never quite reaches.

The function $f(x) = \frac{1}{x}$ has two asymptotes. Describe each of these.

VERTICAL ASYMPTOTE: $x = 0$

HORIZONTAL ASYMPTOTE: $y = 0$

Identify the Domain and Range of this function.

$D: \mathbb{R}, x \neq 0$

$R: \mathbb{R}, y \neq 0$

4. Transformations of $f(x) = \frac{1}{x}$

Use your understanding of transformations to describe each transformation from the parent function $f(x) = \frac{1}{x}$.

a. $y = \frac{1}{x} + 3$ 3 UP	b. $y = \frac{1}{x-2}$ 2 RIGHT	c. $y = -\frac{1}{x}$ REFLECT OVER X-AXIS (OR Y-AXIS)
d. $y = \frac{1}{x+4} - 1$ 4 LEFT 1 DOWN	e. $y = \frac{1}{-x}$ REFLECT OVER Y-AXIS (OR X-AXIS)	f. $y = \frac{-1}{x-5}$ REFLECT OVER X-AXIS 5 RIGHT

5. Draw a rough sketch of each of the following. Identify the Domain and Range of each.

a. $y = \frac{1}{x} + 3$ $D: \mathbb{R}, x \neq 0$ $R: \mathbb{R}, y \neq 3$	b. $y = \frac{1}{x-2}$ $D: \mathbb{R}, x \neq 2$ $R: \mathbb{R}, y \neq 0$	c. $y = -\frac{1}{x}$ $D: \mathbb{R}, x \neq 0$ $R: \mathbb{R}, y \neq 0$
d. $y = \frac{1}{x+4} - 1$ $D: \mathbb{R}, x \neq -4$ $R: \mathbb{R}, y \neq -1$	e. $y = \frac{1}{-x}$ $D: \mathbb{R}, x \neq 0$ $R: \mathbb{R}, y \neq 0$	f. $y = \frac{-1}{x-5}$ $D: \mathbb{R}, x \neq 5$ $R: \mathbb{R}, y \neq 0$