$\frac{8}{\sqrt{3}} \cdot \frac{2}{1} = \frac{16}{\sqrt{3}}$ $\quad \frac{7}{2}$

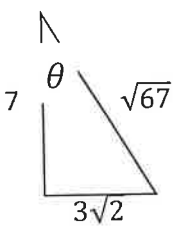
MODULE 11 – TRIGONOMETRIC FUNCTIONS

Revisit Right Triangle Trigonometry (using a table…old school)

θ is many times used as a variable to represent an angle measurement. We will be using θ to represent an unknown angle in these examples.

Practice:
* Write all three trigonometric equations for each triangle using SOH-CAH-TOA. Assume a right triangle.

1.



$\sin\theta = \frac{3\sqrt{2}}{\sqrt{67}}$ $\quad \cos\theta = \frac{7}{\sqrt{67}}$ $\quad \tan\theta = \frac{3\sqrt{2}}{7}$

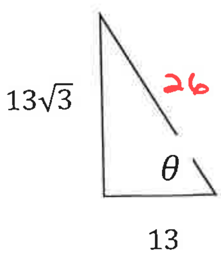
$(7)^2 + (3\sqrt{2})^2 = (\sqrt{67})^2$

$49 + 18 = 67$ TRUE

Also use the Pythagorean Theorem to verify the side lengths of this right triangle.

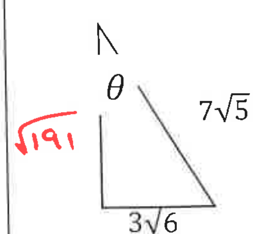
* Write all three trigonometric equations for each triangle using SOH-CAH-TOA. Assume all triangles are right triangles.

2.



$\sin\theta = \frac{13\sqrt{3}}{26}$ $\quad \cos\theta = \frac{13}{26}$ $\quad \tan\theta = \frac{13\sqrt{3}}{13}$

3. This time write the trig. ratios (fractions) as a decimal.



$\sin\theta = \frac{3\sqrt{6}}{7\sqrt{5}}$ .469 $\quad \cos\theta = \frac{\sqrt{191}}{7\sqrt{5}}$ .883 $\quad \tan\theta = \frac{3\sqrt{6}}{\sqrt{191}}$ .532

$a^2 + (3\sqrt{6})^2 = (7\sqrt{5})^2$ $\quad a^2 = 191$

$a^2 + 54 = 245$ $\quad a = \sqrt{191}$

a

Use the trig. table on the next page to identify the angle measurement of θ.

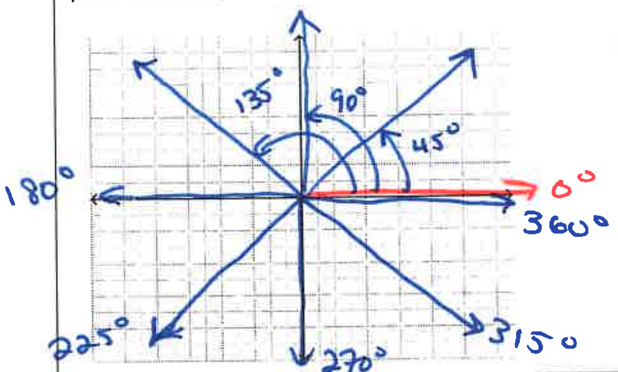
$\theta =$ 28°

Module 11: Lessons 1 & 2 – Angles in Standard Position

Notes:

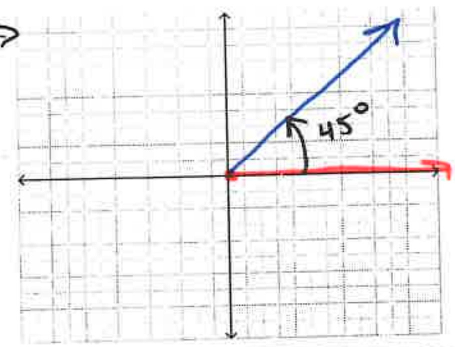
Standard Position: An **angle** is in **standard position** if its vertex is located at the origin and one ray is on the positive x-axis. The ray on the x-axis is called the *initial side* and the other ray is called the *terminal side*.

a. Practice drawing a 45^o angle in standard position with your teacher.

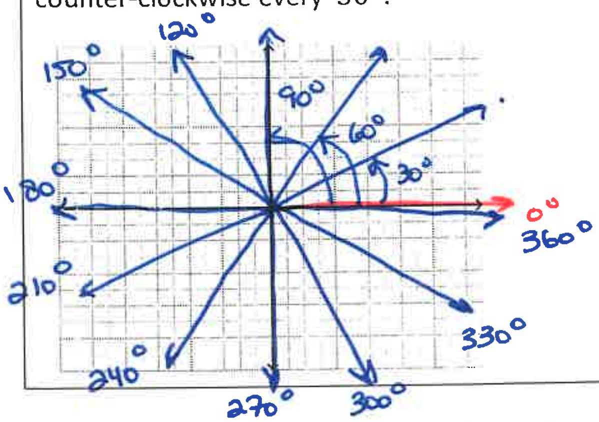


SWITCHED

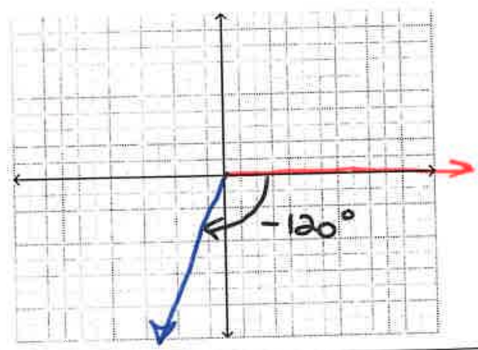
b. Label the terminal rays of each angle as you rotate counter-clockwise every 45^o.



c. Label the terminal rays of each angle as you rotate counter-clockwise every 30^o.

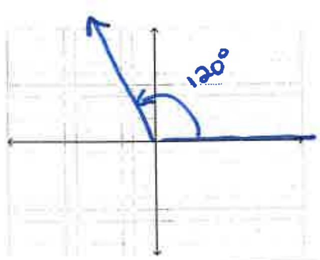


d. Practice drawing a -120^o angle in standard position with your teacher.

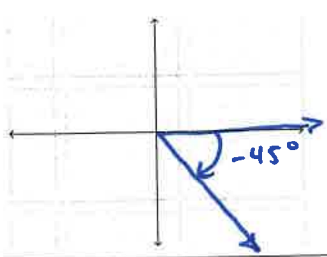


* Practice – Graph each of the following angles in standard position.

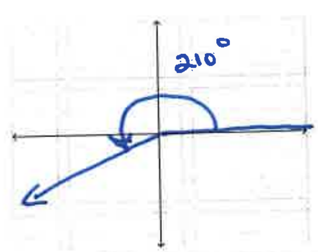
1. 120^o



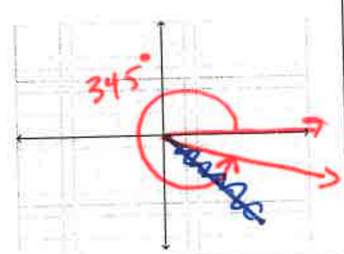
2. -45^o



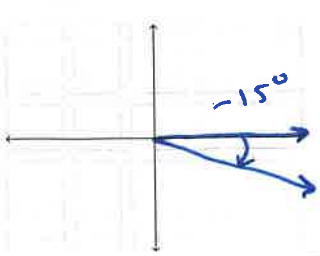
3. 210^o



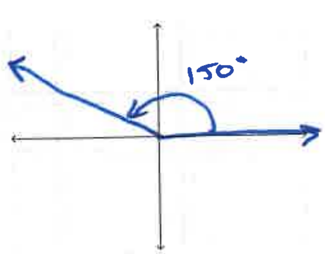
4. 345^o



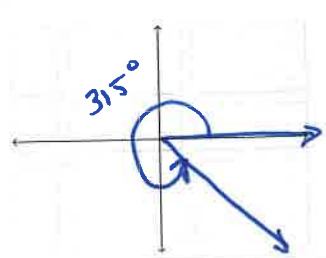
5. -15^o



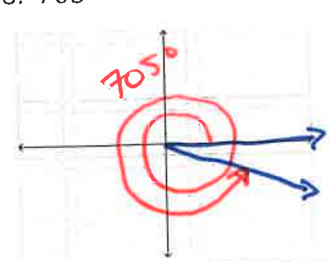
6. 150^o



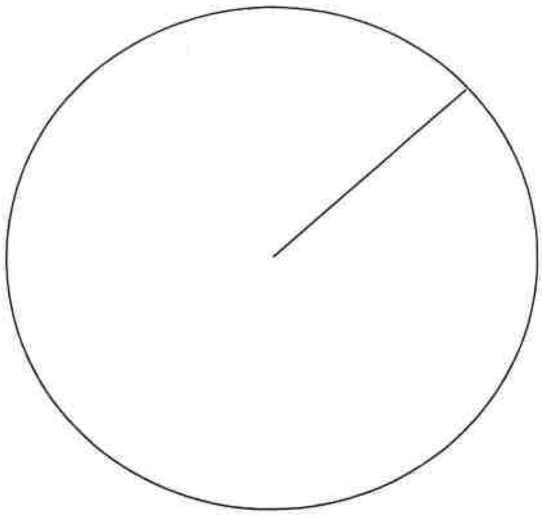
7. 315^o



8. 705^o



What is a radian?

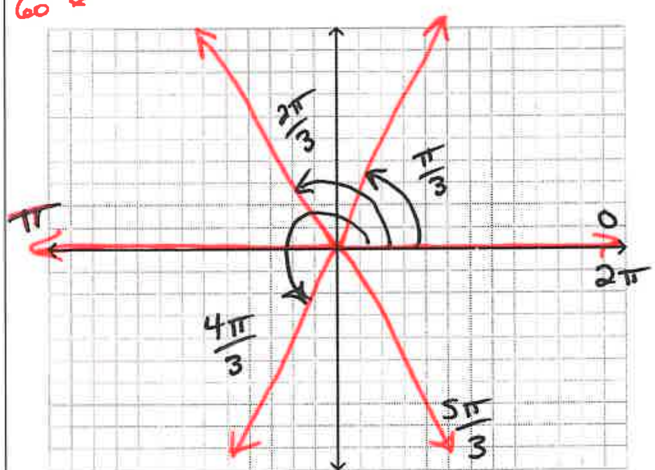


IN CLASS DISCUSSION

Angles in Standard Position (Radians)

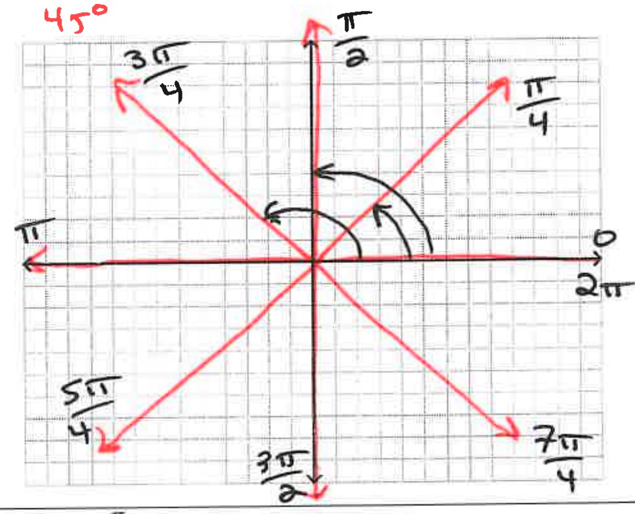
Count by $\frac{\pi}{3}$ for one full rotation.

60°

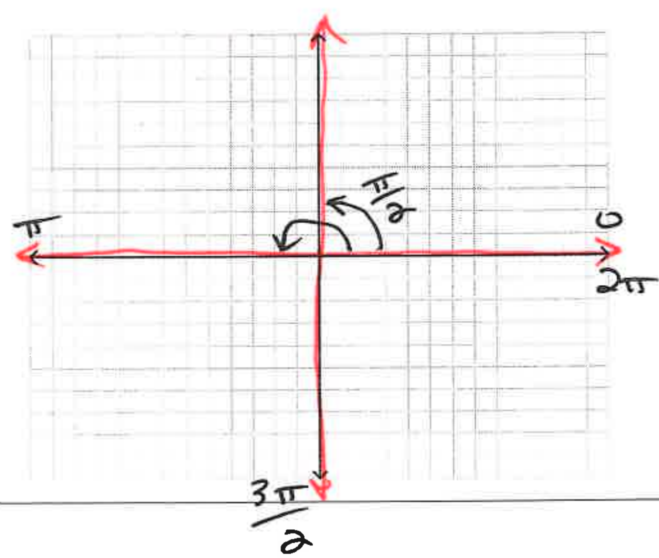


Count by $\frac{\pi}{4}$ for one full rotation.

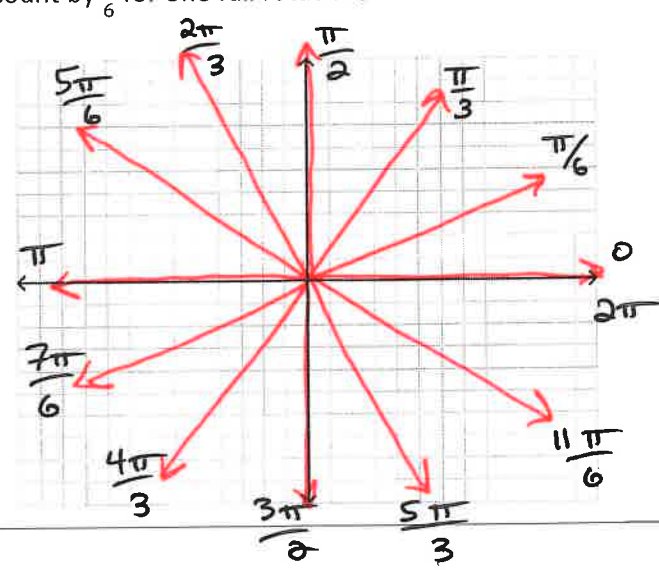
45°



Count by $\frac{\pi}{2}$ for one full rotation.



Count by $\frac{\pi}{6}$ for one full rotation.



Notes:

Conversion Bank

1 inch = 2.54 cm	1 mile = 1.61 kilometers	2π radians = 360 degrees
1 barrel = 31.5 gallons	1 gallon = 3.7854 liters	π radians = 180 degrees

* Convert each of the following.

1. How many degrees are in 4.4π radians? $\frac{4.4\pi \text{ RADIANS}}{1} \cdot \frac{180^\circ}{\pi \text{ RADS}} = 792^\circ$	2. How many meters are in 100 yards? $\frac{100 \text{ YDS}}{1} \cdot \frac{36 \text{ IN}}{1 \text{ YD}} \cdot \frac{2.54 \text{ cm}}{1 \text{ IN}} \cdot \frac{1 \text{ m}}{100 \text{ cm}} = 91.44 \text{ m}$
3. How many gallons are in a 2-liter bottle of soda? $\approx .53$ GAL	4. How many miles are in a 5K race? ≈ 3.1 MILES

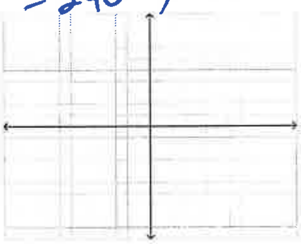
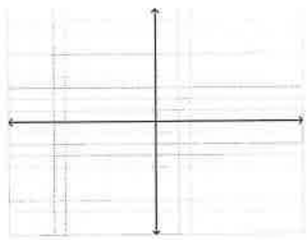
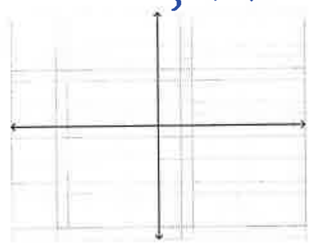
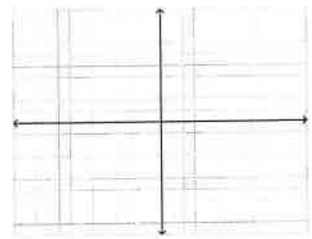
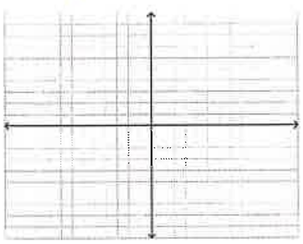
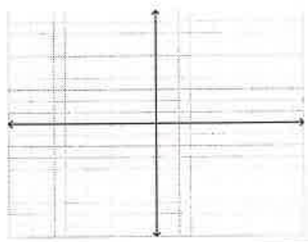
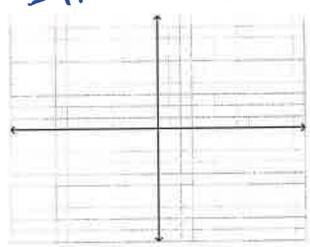
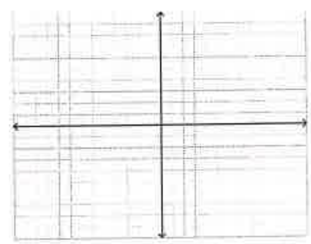
Convert each angle measurement in radians to degrees.

5. $\frac{\pi}{7}$ radians $\approx 25.7^\circ$	6. $\frac{\pi}{5}$ radians 36°	7. $\frac{\pi}{12}$ radians 15°
8. $.8\pi$ radians 144°	9. 7 radians $\approx 401.1^\circ$	10. 3π radians 540°

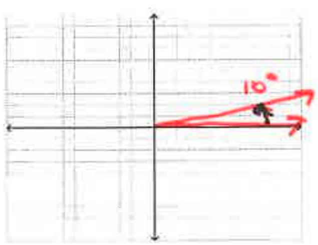
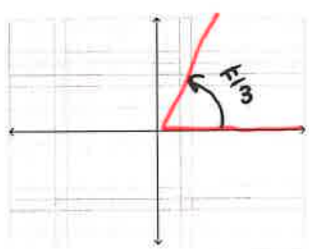
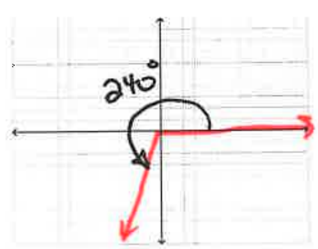
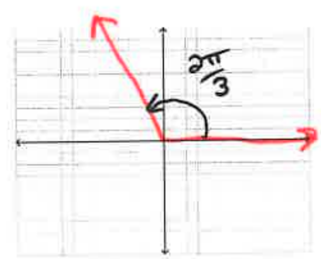
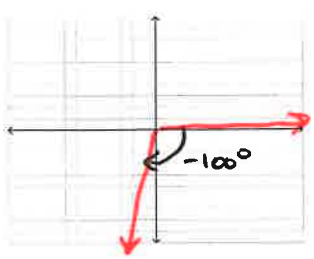
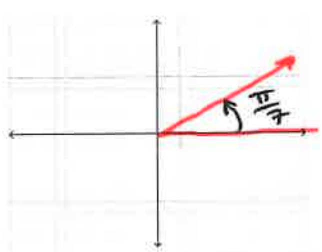
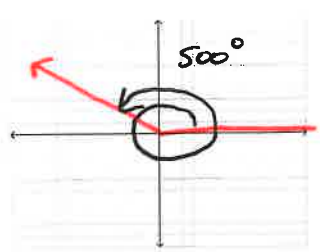
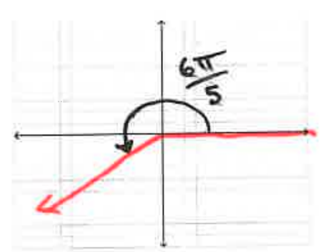
Coterminal Angles - angles in standard position (initial ray on the positive x-axis) that have a common terminal ray.

many correct solutions. here are some.

Identify at least two coterminal angles for each angle. If the stated angle is in degrees, give your answers in degrees. If the angle is in radians, give your answers in radians.

1. 120^o -240°, 480° 	2. $-\frac{\pi}{4}$ $\frac{7\pi}{4}$, $\frac{-9\pi}{4}$ 	3. 210^o 570°, -150° 	4. π $-\pi$, 3π 
5. -15^o 705° -375° 	6. $\frac{4\pi}{3}$ -45° -405° 	7. 315^o -1125° 2115° 	8. $\frac{3\pi}{4}$ $\frac{11\pi}{4}$ $-\frac{5\pi}{4}$ 

Graph each of the following angles in standard position. Be accurate even though you are estimating the angle.

1. 10^o  	2. $\frac{\pi}{3}$  	3. 240^o  	4. $\frac{2\pi}{3}$ 
5. -100^o  	6. $\frac{\pi}{7}$  	7. 500^o  	8. $\frac{6\pi}{5}$ 

Coterminal Angles - *Coterminal angles* are angles in standard position (angles with the initial side on the positive x-axis) that have a common terminal side.

9. Identify at least two coterminal angles for each angle in problems 1-8 on the previous page.	10. Identify two other angles measurements that are coterminal with a 30^o angle in standard position.

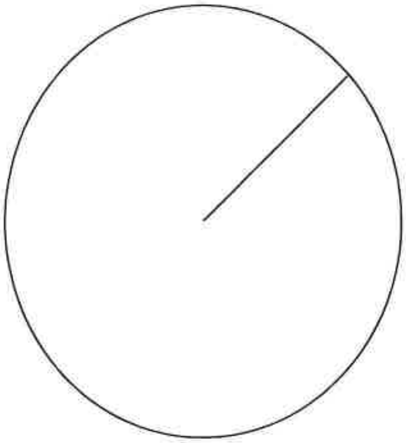
Conversions (including radians & degrees)

Notes:

Conversions – Many times when you do not know a direct conversion from one unit to the next, you can use unit conversions. (*This is an organized way of working through conversions.*)

Converting Radians to Degrees –

Looking at the circle and radius drawn on the right; if you were to take that radius and bend it around the circle, how many of those lengths do think would fit around the circle?



Your Estimate ________

Actual Value ________

Therefore, there are ________ radians per revolution or every 360 degrees.

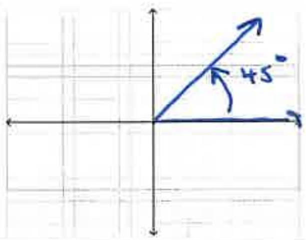
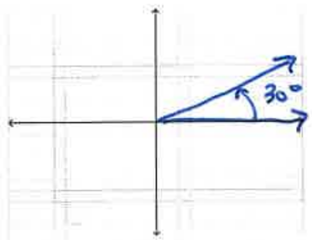
Fractions to use when converting radians and degrees

---

* Convert each from radians to degrees or degrees to radians.

Ex. 4: 2.5π radians	Ex. 5: 60 degrees	Ex. 6: 2 radians	Ex. 7: $150°$
$450°$	$\pi/3$	$\approx 114.6°$	$5\pi/6$

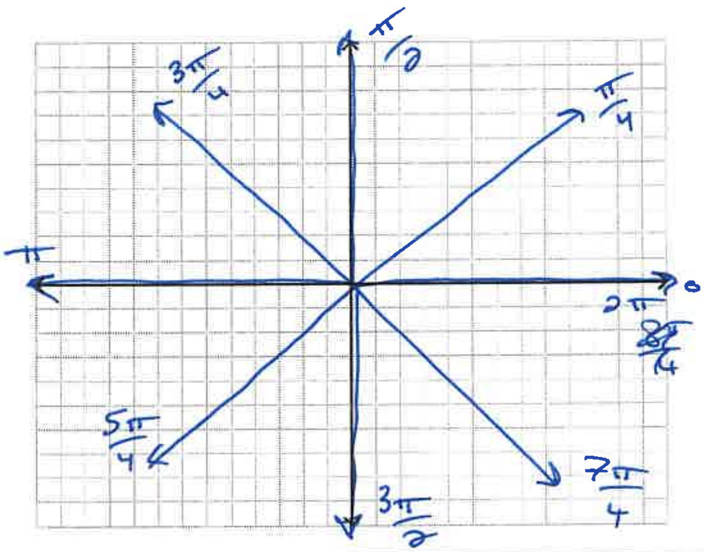
* Review –

1. Graph 45^o in standard position.	2. Graph 30^o in standard position.	3. Convert 30^o to radians.	4. Convert 45^o to radians.
		$\frac{\pi}{6}$ RADS	$\frac{\pi}{4}$ RADS

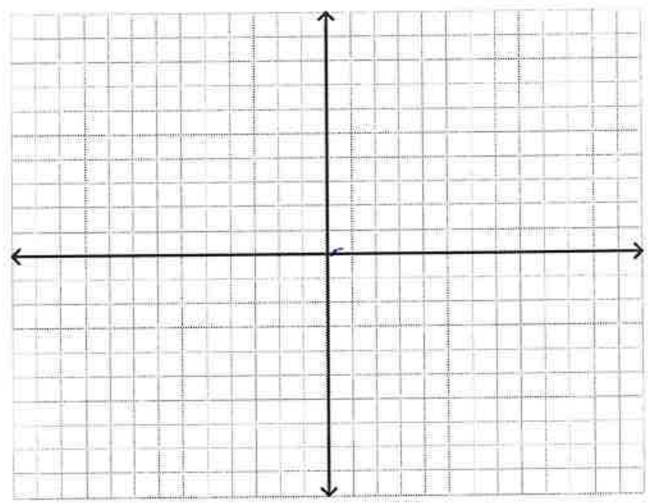
SEE PAGE 4

Notes:

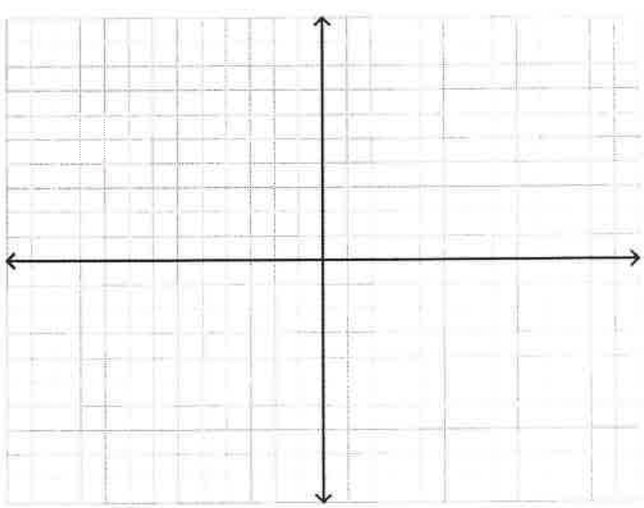
a. Label the terminal rays of each angle as you rotate counter-clockwise every $\frac{\pi}{4}$ radians.



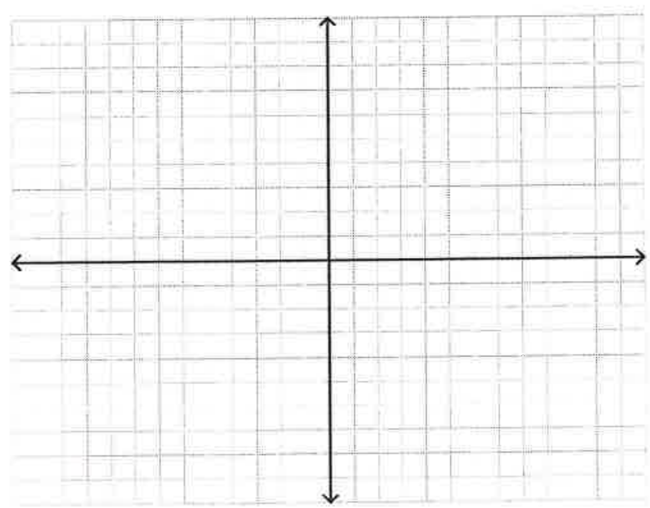
b. Label the terminal rays of each angle as you rotate counter-clockwise every $\frac{\pi}{6}$ radians.



c. Label the terminal rays of each angle as you rotate counter-clockwise every $\frac{\pi}{3}$ radians.



d. Label the terminal rays of each angle as you rotate counter-clockwise every $\frac{\pi}{8}$ radians.



10

Reciprocal Trigonometric Functions

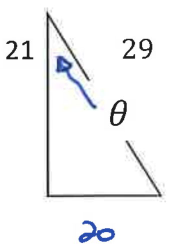
Quick Review of SOH-CAH-TOA

These Trigonometric Functions also have reciprocals.

The cosecant: $\frac{1}{\sin\theta} = \csc\theta$	The secant: $\frac{1}{\cos\theta} = \sec\theta$	The cotangent: $\frac{1}{\tan\theta} = \cot\theta$

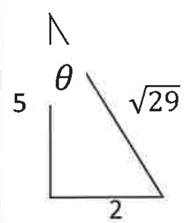
Assume each triangle is a right triangle. Write all three trigonometric equations for each triangle using SOH-CAH-TOA.

1.



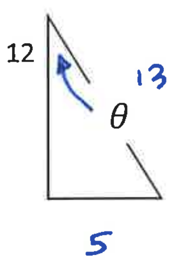
$\sin\theta = \frac{20}{29}$	$\csc\theta = \frac{29}{20}$
$\cos\theta = \frac{21}{29}$	$\sec\theta = \frac{29}{21}$
$\tan\theta = \frac{20}{21}$	$\tan\theta = \frac{21}{20}$

2.



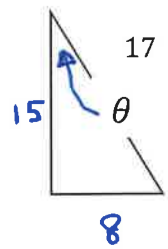
$\sin\theta = \frac{2}{\sqrt{29}}$	$\csc\theta = \frac{\sqrt{29}}{2}$
$\cos\theta = \frac{5}{\sqrt{29}}$	$\sec\theta = \frac{\sqrt{29}}{5}$
$\tan\theta = \frac{2}{5}$	$\tan\theta = \frac{5}{2}$

3.



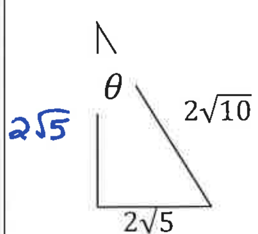
$\sin\theta = \frac{5}{13}$	$\csc\theta = \frac{13}{5}$
$\cos\theta = \frac{12}{13}$	$\sec\theta = \frac{13}{12}$
$\tan\theta = \frac{5}{12}$	$\tan\theta = \frac{12}{5}$

4.



$\sin\theta = \frac{8}{17}$	$\csc\theta = \frac{17}{8}$
$\cos\theta = \frac{15}{17}$	$\sec\theta = \frac{17}{15}$
$\tan\theta = \frac{8}{15}$	$\tan\theta = \frac{15}{8}$

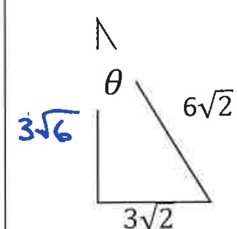
5.



$\frac{2\sqrt{5}}{2\sqrt{10}}$

$\sin\theta = \frac{2\sqrt{5}}{2\sqrt{10}} = \frac{1}{\sqrt{2}}$	$\csc\theta = \sqrt{2}$
$\cos\theta = \frac{1}{\sqrt{2}}$	$\sec\theta = \sqrt{2}$
$\tan\theta = 1$	$\tan\theta = 1$

6.



$\frac{3\sqrt{2}}{6\sqrt{2}}$

$\frac{3\sqrt{6}}{6\sqrt{2}}$

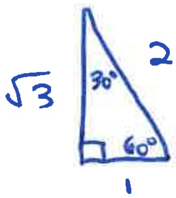
$\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{1}{2}$	$\csc\theta = 2$
$\cos\theta = \frac{\sqrt{3}}{2}$	$\sec\theta = \frac{2}{\sqrt{3}}$
$\tan\theta = \frac{1}{\sqrt{3}}$	$\tan\theta = \sqrt{3}$

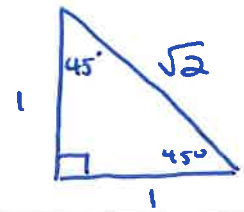
$\frac{3\sqrt{2}}{3\sqrt{6}}$

Trigonometry in the First Quadrant

Reference Triangle for 30^o and 60^o.



Reference Triangle for 45^o.



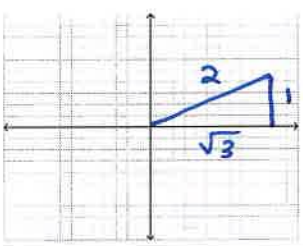
Notes:

To Evaluate a trig. expression without a calculator:

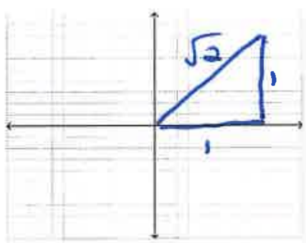
1. Draw the angle in standard position.
2. Create a "Reference" Right Triangle by dropping an altitude from the terminal ray to the x-axis.
3. Use a Special Right Triangle and a Trigonometric Ratio to determine the value.

* Practice – Evaluate each trig. expression without a calculator.

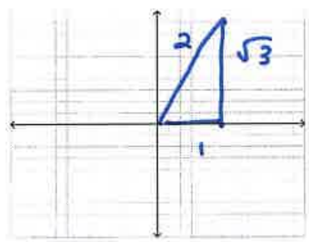
1. $\sin 30^o = \frac{1}{2}$



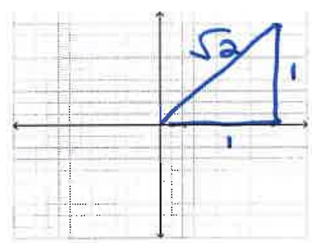
2. $\cos 45^o = \frac{1}{\sqrt{2}}$



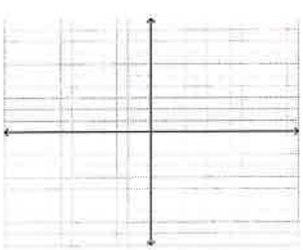
3. $\tan 60^o = \sqrt{3}$



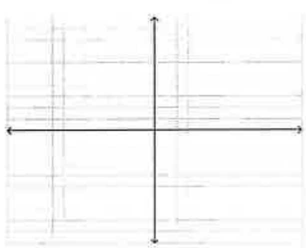
4. $\sin 45^o = \frac{1}{\sqrt{2}}$



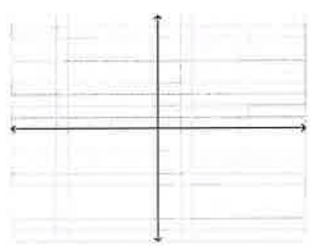
5. $\tan 45^o = 1$



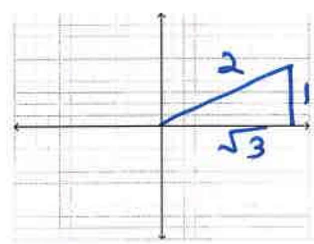
6. $\sin 60^o = \frac{\sqrt{3}}{2}$



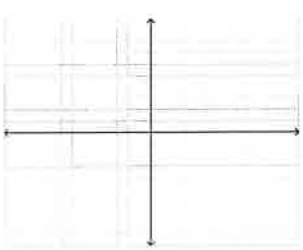
7. $\cos 60^o = \frac{1}{2}$



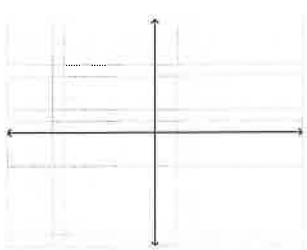
8. $\cos 30^o = \frac{\sqrt{3}}{2}$



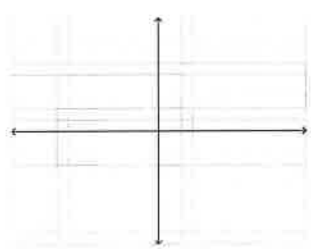
9. $\tan 30^o = \frac{1}{\sqrt{3}}$



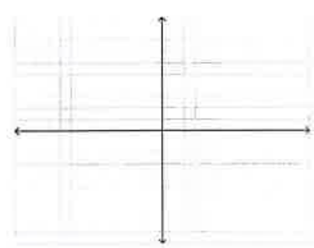
10. $\csc 60^o = \frac{2}{\sqrt{3}}$



11. $\sec 45^o = \sqrt{2}$

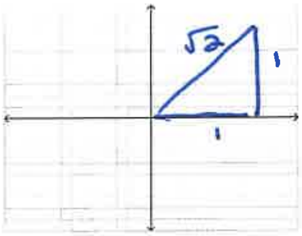


12. $\cot 30^o = \sqrt{3}$

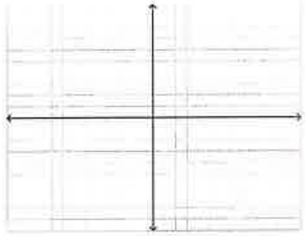


* Practice –Draw each reference triangle and evaluate the expression.

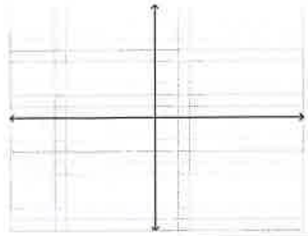
1. $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$



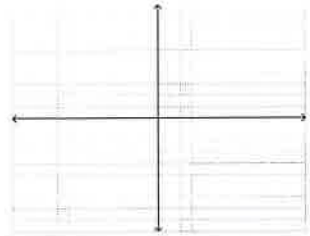
2. $\cos\frac{\pi}{3} = \frac{1}{2}$



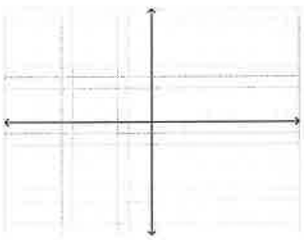
3. $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$



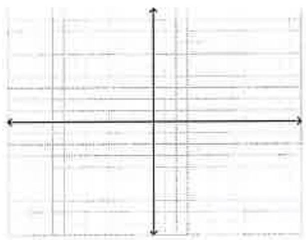
4. $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$



5. $\tan\frac{\pi}{4} = 1$

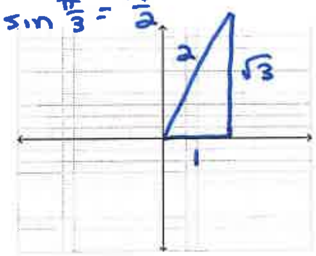


6. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

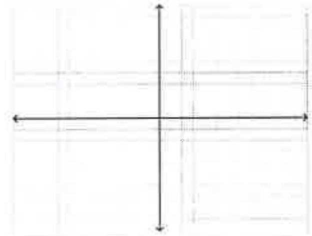


7. $\csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$



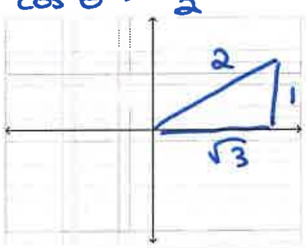
8. $\sec\frac{\pi}{4} = \sqrt{2}$



Solve for values of θ between $0°$ *and* $90°$ (*quadrant I*).

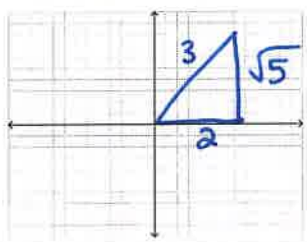
1. *If* $\sin\theta = \frac{1}{2}$, *find* $\cos\theta$

$\cos\theta = \frac{\sqrt{3}}{2}$



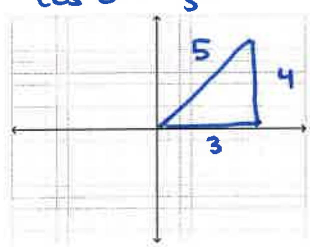
2. *If* $\cos\theta = \frac{2}{3}$, *find* $\sin\theta$

$\sin\theta = \frac{\sqrt{5}}{3}$



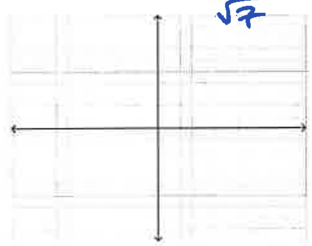
3. *If* $\sin\theta = \frac{4}{5}$, *find* $\cos\theta$

$\cos\theta = \frac{3}{5}$



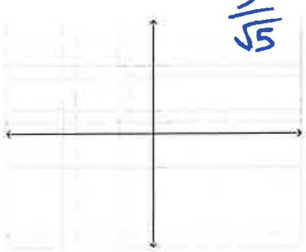
4. *If* $\sin\theta = \frac{3}{4}$, *find* $\sec\theta$

$\frac{4}{\sqrt{7}}$



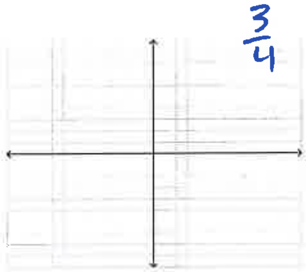
5. *If* $\cos\theta = \frac{2}{3}$, *find* $\csc\theta$

$\frac{3}{\sqrt{5}}$



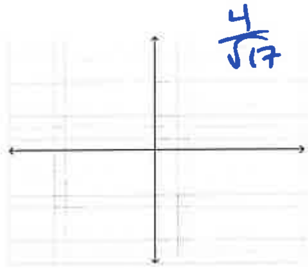
6. *If* $\cos\theta = \frac{4}{5}$, *find* $\tan\theta$

$\frac{3}{4}$



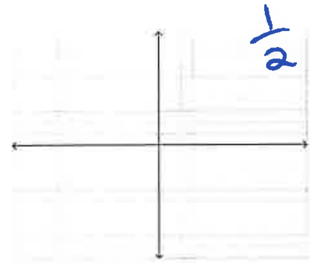
7. *If* $\tan\theta = 4$, *find* $\sin\theta$

$\frac{4}{\sqrt{17}}$



8. *If* $\cot\theta = 2$, *find* $\tan\theta$

$\frac{1}{2}$



Trigonometry in All Quadrants

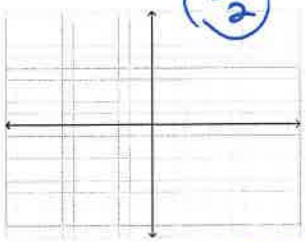
Notes:

To Evaluate a trig. expression without a calculator:

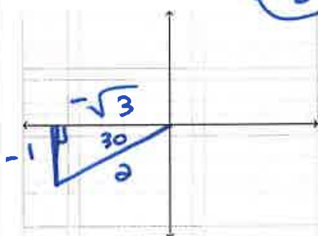
1. Draw the angle in standard position.
2. Create a "Reference" Right Triangle by dropping an altitude from the terminal ray to the x-axis.
3. Use a Special Right Triangle and a Trigonometric Ratio to determine the value.
4. **Pay Close Attention to Positive and Negative Values.**

* Practice – Evaluate each trig. expression without a calculator.

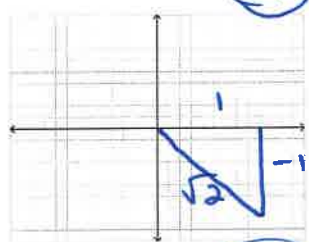
1. $\sin 30^o$ $\frac{1}{2}$



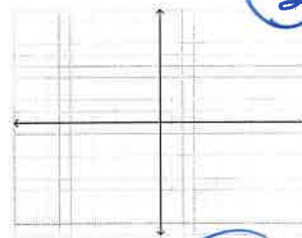
2. $\cos 210^o$ $= -\frac{\sqrt{3}}{2}$



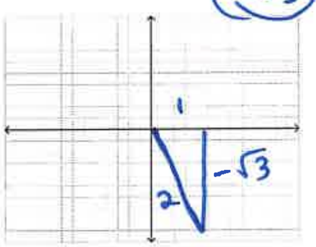
3. $\tan 315^o$ $= -1$



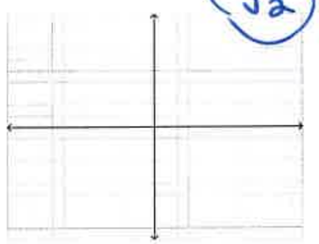
4. $\sin 150^o$ $= \frac{1}{2}$



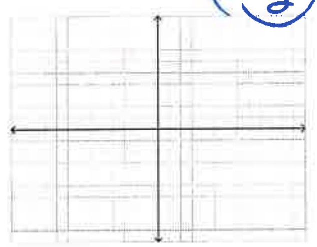
5. $\tan 300^o$ $= -\sqrt{3}$



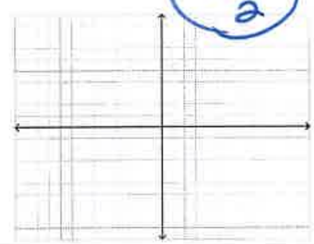
6. $\sin 225^o$ $= -\frac{1}{\sqrt{2}}$



7. $\cos 120^o$ $= -\frac{1}{2}$

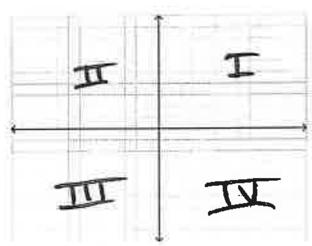


8. $\cos 390^o$ $\frac{\sqrt{3}}{2}$



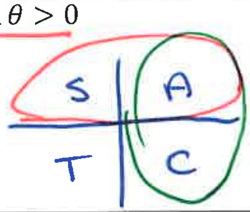
What do you notice about the signs of each trigonometric function in each quadrant?

IN CLASS DISCUSSION

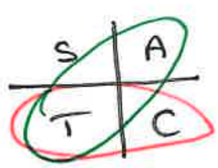


Determine the quadrant.

1. $\cos\theta > 0, \sin\theta > 0$ — I



2. $\tan\theta > 0, \sin\theta < 0$ — III



3. $\cot\theta < 0, \sin\theta < 0$ — IV

4. $\tan\theta < 0, \cos\theta < 0$ — II

5. $\csc\theta < 0, \cos\theta < 0$ — III

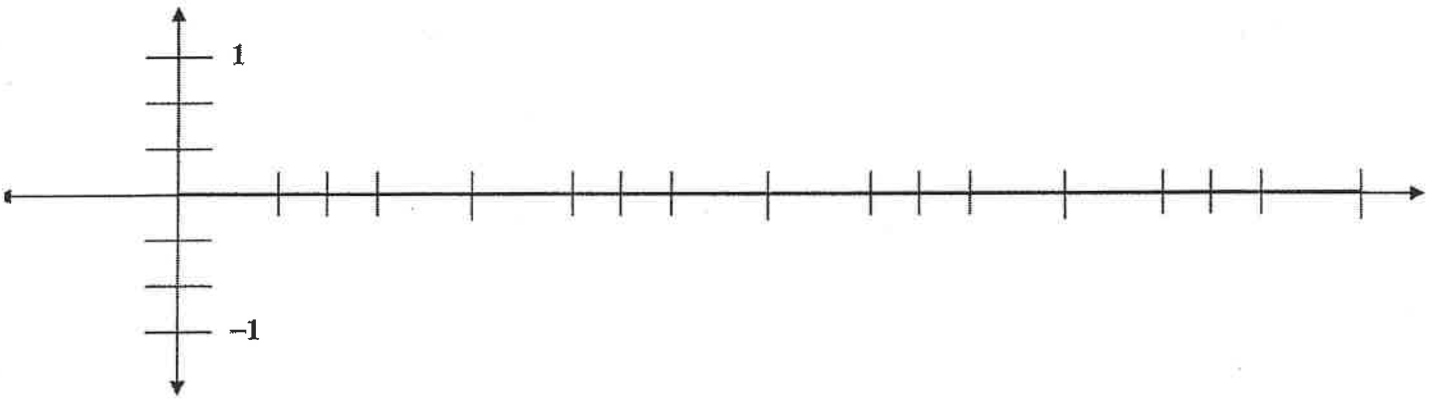
6. $\sec\theta > 0, \cot\theta > 0$ — I

Module 11: Lessons 3 – Graphing Sine and Cosine Functions

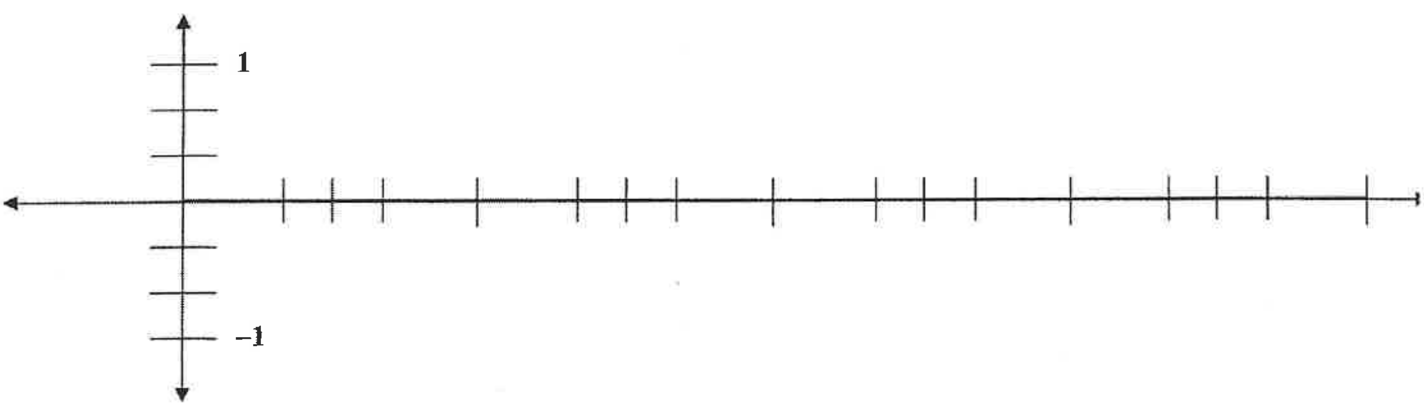
IN CLASS DISCUSSION

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0				1				0				-1				0
$y = cos\, x$	1				0				-1				0				1

Graph: $y = \sin x$. The sine graph is smooth and rounded. Use the points from the t-table to help sketch its graph.



Graph: $y = \cos x$. The cosine graph is smooth and rounded. Use the points from the t-table to help sketch its graph.



Streamline the Graphing Process (One Period)

$y = \sin(x)$

$y = cos(x)$

Matching Sine Wave Functions

CURVE FITTING

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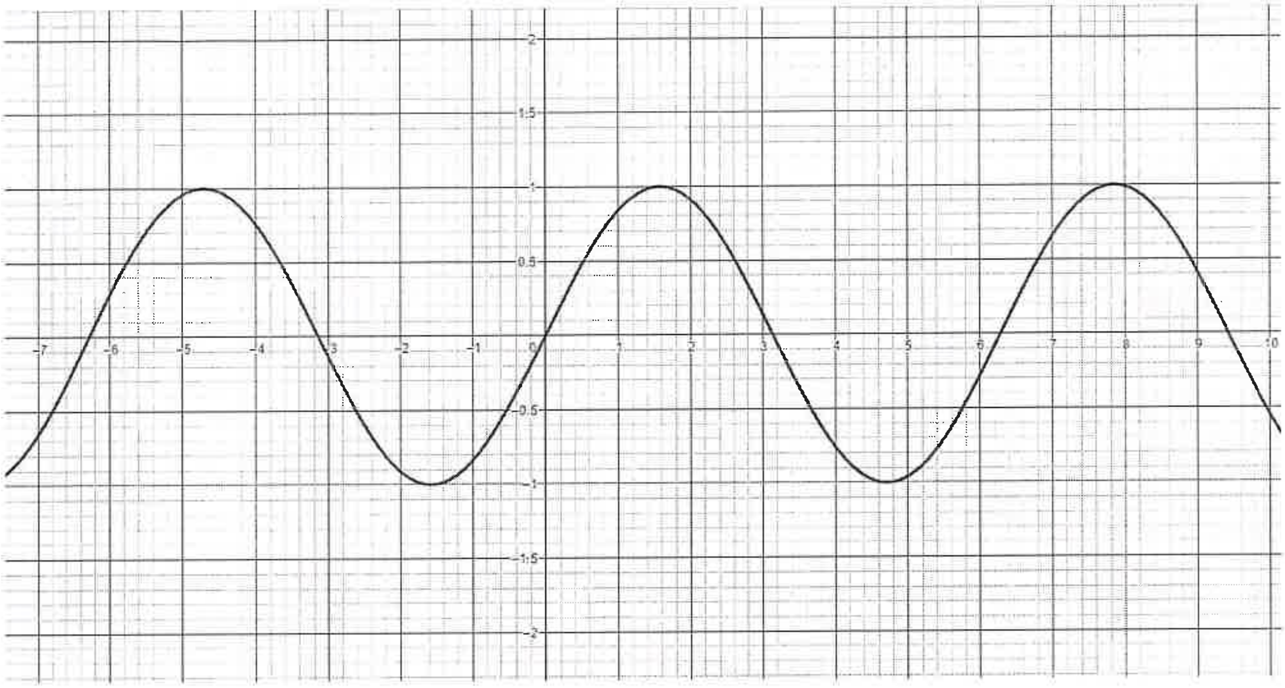
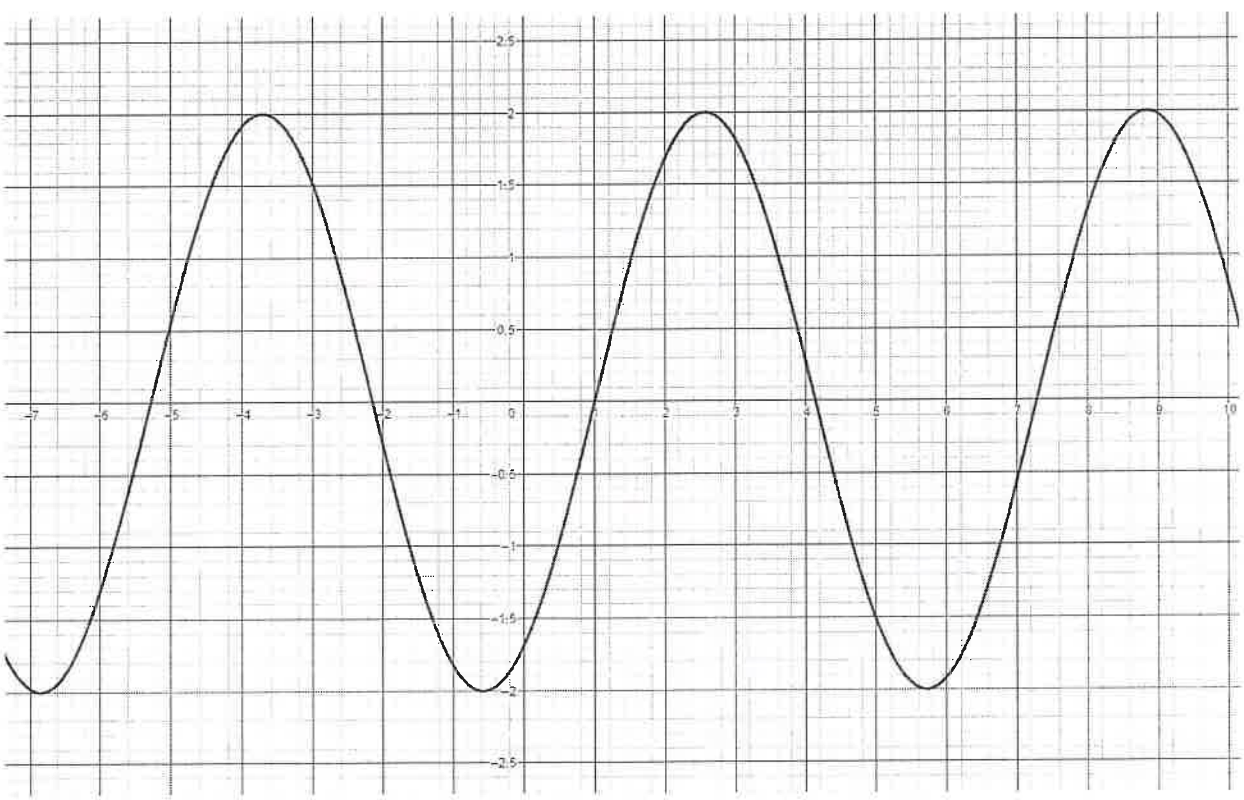
Reference: Sine Wave Parent Function $y = \sin(x)$



Duplicate each wave form using the parent function $y = \sin(x)$ and your understanding of transformations.
Write your function equation in the space provided.

1.



Duplicate each wave form using the parent function $y = \sin(x)$ and your understanding of transformations.

Write your function equation in the space provided.

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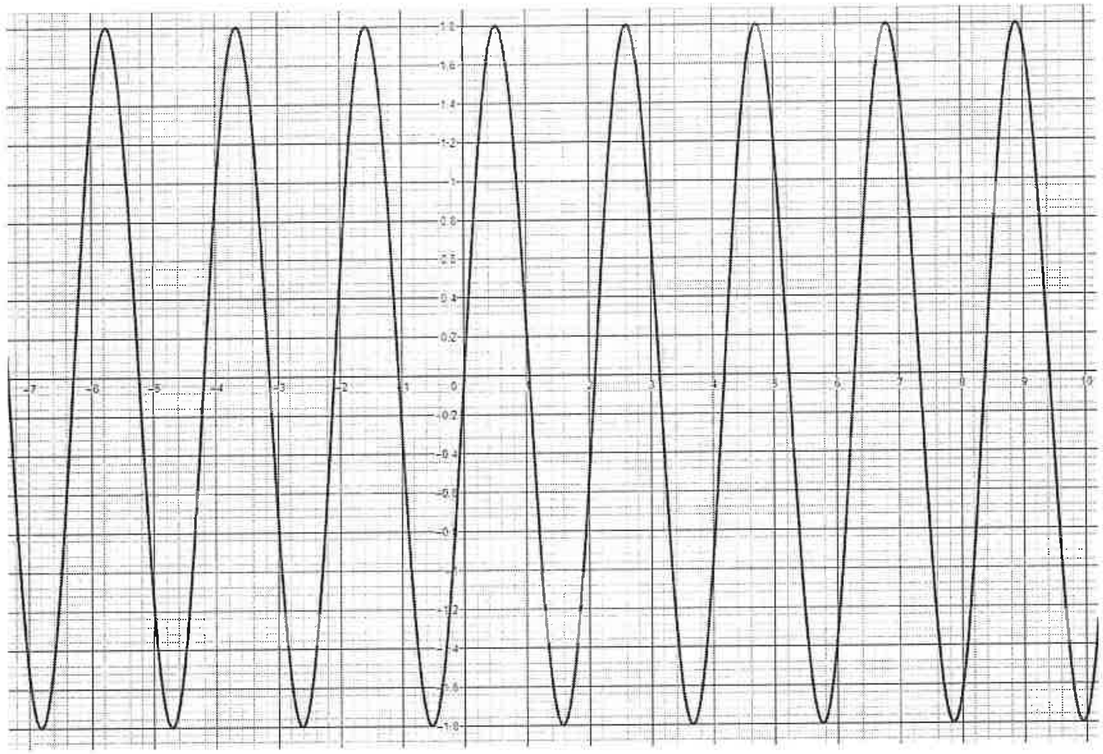
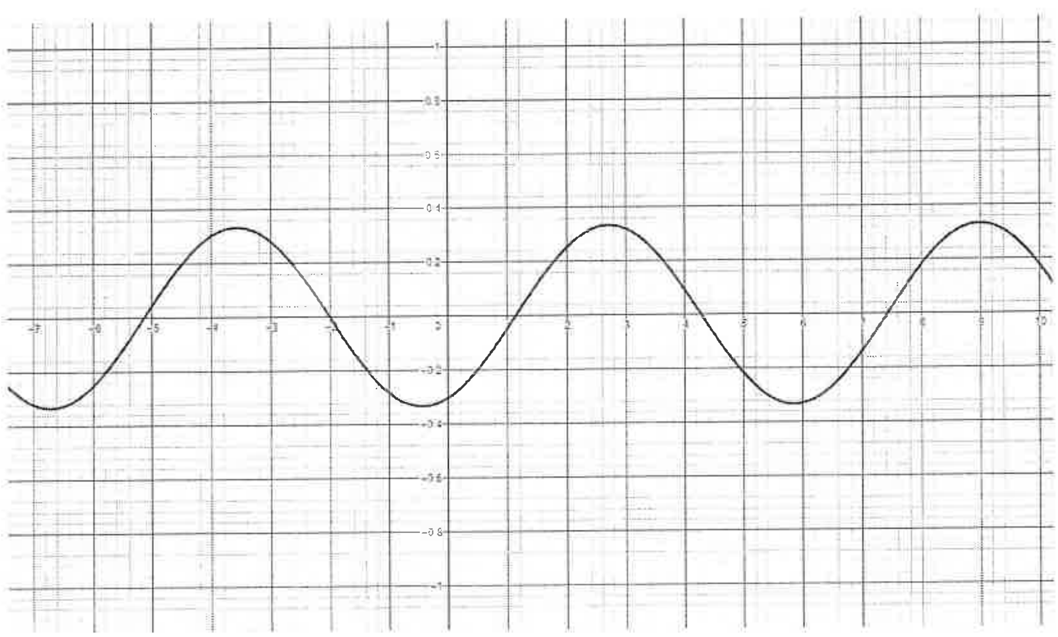
2.



Duplicate each wave form using the parent function $y = \sin(x)$ and your understanding of transformations.
Write your function equation in the space provided.

3.



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r. Find the following information for the function: $y = -\sin\left(\frac{x}{3} - \pi\right) - 4$. $NOTE: -\sin\left(\frac{x}{3}-\pi\right)-4 = -\sin\frac{1}{3}(x-3\pi)-4$

Amplitude: __________ Domain: __________

Period: __________ Range: __________

Phase Shift: __________ Appropriate interval to graph one complete wave: __________

Vertical Shift: __________ y-intercept: __________

s. Find the following information for the function: $y = 3\cos\left(2\pi x + \frac{\pi}{3}\right) + 2$. $NOTE: 3\cos\left(2\pi x+\frac{\pi}{3}\right)+2 = 3\cos 2\pi\left(x+\frac{1}{6}\right)+2$

Amplitude: __________ Domain: __________

Period: __________ Range: __________

Phase Shift: __________ Appropriate interval to graph one complete wave: __________

Vertical Shift: __________ y-intercept: __________