MODULE 6: INVERSES & RADICAL FUNCTIONS

Lesson 1: Operations on Functions (Please see online book for additional examples.)

Given:	$f(x) = x^2 + 1$	$g(x) = x + 5$	$h(x) = -x - 3$	$p(x) = x^2 + 4x$	$r(x) = 6x - 1$

Evaluate each.		
1. $r(3)$	2. $p(-2)$	3. $h(22)$
4. $g(-10)$	5. $f(5)$	6. $f(-5)$
Perform each operation and simplify the expression. State any domain restrictions when necessary.		
7. $(h + p)(x)$	8. $(g - r)(x)$	9. $(f \cdot r)(x)$
10. $(g - h)(x)$	11. $\left(\frac{h}{g}\right)(x)$	12. $\left(\frac{r}{f}\right)(x)$
Evaluate and Simplify each Composite Function.		
13. $g(f(3))$	14. $h(r(5))$	15. $f(p(-1))$
16. $(h\ o\ g)(-2)$	17. $(r\ o\ f)(3)$	18. $g(f(x))$
19. $g(p(x))$	20. $p(g(x))$	21. $h(r(x))$

Module 6: Lesson 1 – Example 3

Graphs of Combined Functions

POPULATION The population in millions of India t years after 1955 can be modeled by the linear function $f(t) = 15.7t + 344.2$, and the population of the United States in millions t years after 1955 can be modeled by $g(t) = 2.6t + 165.8$. Define and graph the function that represents how much greater India's population is than that of the U.S. by year.

Notes:

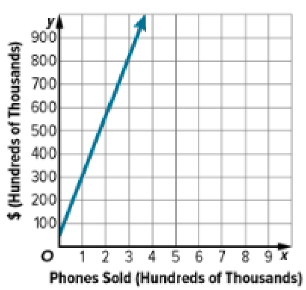
Module 6: Lesson 1 – Extra Example 3

Graphs of Combined Functions

PROFIT A tech company produces phones, earning a revenue $r(x) = 200x$, where x is the number of phones produced and sold. The cost is $c(x) = 60x + 5{,}000{,}000$. Graph the function that represents the profit $P(x)$ the company earns when x phones are sold.

Select the graph of $P(x)$.

○ A



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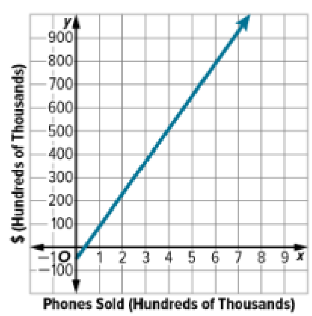

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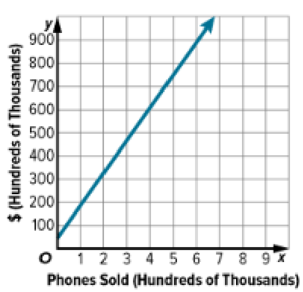

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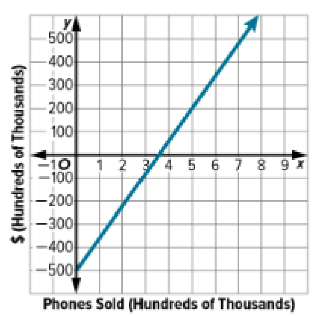


Notes:

Evaluating Functions

$$f(x) = 3x^2 - 7$$

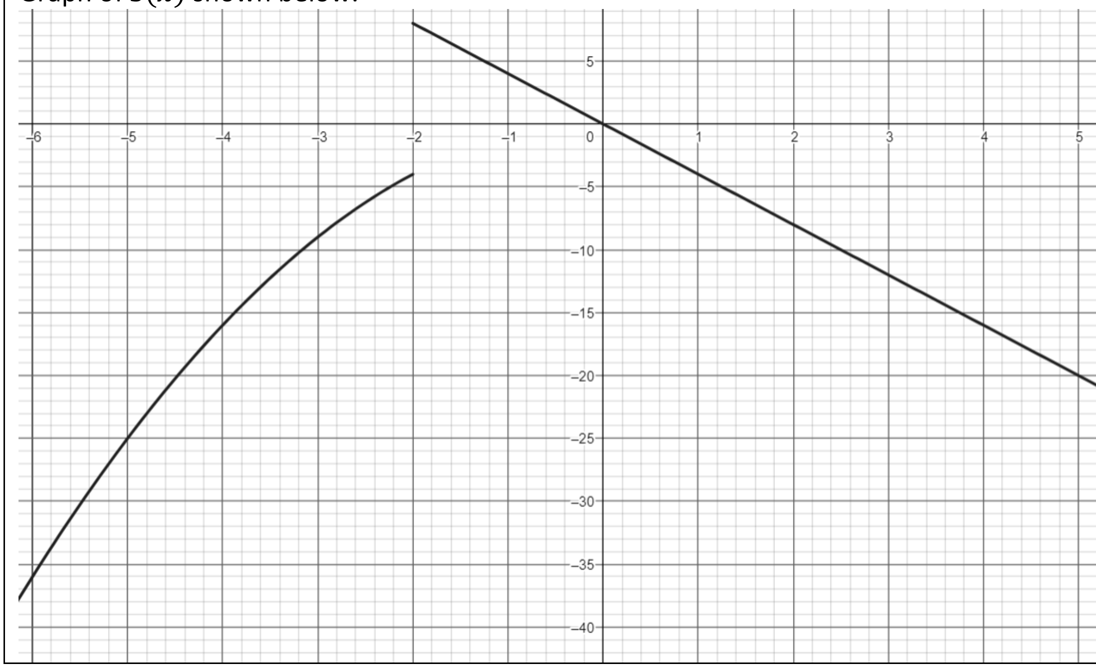
$$g(x) = \sqrt{x-3}$$

$$h(x) = 2x^3$$

x	$r(x)$
-1	1.5
0	3
1	6
2	12
3	24
4	96

$$p(x) = \begin{cases} -x & \text{if} \quad -30 \le x < 1 \\ 3x+4 & \text{if} \quad 1 \le x \le 7 \\ \frac{1}{2}x + 1 & \text{if} \quad x > 7 \end{cases}$$

Graph of $s(x)$ shown below.



Determine each value.

1. $g(19)$	2. $r(2)$	3. $s(5)$	4. $f(x) = 68$ Find x.	5. $f(-4)$
6. $h(x) = -54$ Find x.	7. $p(s(-5))$	8. $s(x) = -25$ Find x.	9. $f(h(1))$	10. $r(x) = 24$ Find x.
11. $g(x) = 52$ Find x.	12. $p(-5)$	13. $s(x) = 4$ Find x.	14. $h(x) = 250$ Find x.	15. $s(g(7))$
16. $g(p(8))$	17. $f(x) = 209.75$ Find x.	18. $r(g(12))$	19. Challenge $p(x) = 35$ Find x.	20. Challenge $p(x) = 4$ Find x.

Introduction to the Difference Quotient

<u>Don't be afraid, but this is the basic concept of the Derivative which is used in Calculus to look at Rates of Change.</u>

Warm-Up

Evaluate and Simplify each when $f(x) = x^2 - 5x$.

1. $f(3)$	2. $f(7)$	3. $f(-4)$	4. $f(1)$
5. $f(x)$	6. $f(a)$	7. $f(f(3))$	8. $f(f(-2))$
9. $f(a+h)$	10. $f(f^{-1}(x))$	11. $f(b+5)$	12. $f(x^2)$

You should recognize this: $$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$	Similar to this what is titled the Difference Quotient $$\boldsymbol{m = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}}$$

13. Evaluate and Simplify the Difference Quotient for $f(x) = x^2 - 5x$.	14. Evaluate and Simplify the Difference Quotient for $g(x) = 2x^2 + 3$.

Module 6: Lesson 2 - Inverse Functions (Please see online book for additional examples.)

Algebraically determine the inverse of each function. Write the inverse function with correct notation.
Check for correctness either using graphs, tables, or compositions.

1. $f(x) = 2x - 4$	2. $g(x) = (x - 7)^2$	3. $h(x) = x^2 + 6x + 5$
4. $p(x) = \sqrt{2x - 3}$	5. $r(x) = \frac{1}{2}x - 3$	6. $s(x) = 4\sqrt{x - 5}$
7. $k(x) = x^2 - 8x + 1$	8. $l(x) = 7x + 6$	9. $q(x) = \sqrt{2x + 9}$

Module 6: Lesson 3 & 5 – nth Roots & Rational Exponents & Radical Operations(Please see online book for additional examples.)

Simplify each of the following Radical Expressions. Assume all variables are positive.

1. $\sqrt{50}$	2. $\sqrt{32x^5}$	3. $\sqrt[3]{27b^3c^4}$

* Fill a value into each radical so that the result is a whole number solution, then write the solution. A couple of problems are already filled in as examples.

4. $\sqrt[2]{121} = 11$	5. $\sqrt{\_\_\_\_} = \_\_\_\_\_\_$	6. $\sqrt[2]{\_\_\_\_} = \_\_\_\_\_\_$
7. $\sqrt[3]{64} = 4$ because $4 \cdot 4 \cdot 4 = 64$	8. $\sqrt[3]{\_\_\_\_} = \_\_\_\_\_\_$	9 $\sqrt[4]{\_\_\_\_} = \_\_\_\_\_\_$

* Review – Rationalize the denominator in each of the following. Again, the first is provided as an example.

Rationalizing the denominator – writing an equivalent value without the root in the denominator

10. $\frac{5}{\sqrt[2]{3}}$ $\frac{5}{\sqrt[2]{3}} \cdot \frac{\sqrt[2]{3}}{\sqrt[2]{3}} = \frac{5\sqrt[2]{3}}{\sqrt[2]{9}} = \frac{5\sqrt[2]{3}}{3}$	11. $\frac{5}{\sqrt[2]{2}}$	12. $\frac{5}{\sqrt{7}}$

* Rationalize the denominator in each.

13. $\frac{8}{\sqrt[3]{2}}$ $\frac{8}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{8\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{8\sqrt[3]{4}}{2} = 4\sqrt[3]{4}$	14. Written Response: Why did the example at the left multiply by one in the form of $\frac{\sqrt[3]{4}}{\sqrt[3]{4}}$?

15. $\frac{7}{\sqrt[3]{2}}$	16. $\frac{12}{\sqrt[3]{9}}$

OPERATIONS WITH RADICAL EXPRESSIONS * Intro/Review – Working with Rational Exponents	Example: $(8)^{\frac{2}{3}}$ means $\left(\sqrt[3]{8}\right)^2$. Do you see where the 2 and the 3 from the rational exponent are in radical expression?
A Rational Exponent is a Root and a Power Combined.	So $(8)^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

Simplify each of the following Radical Expressions.

1. $(27)^{\frac{2}{3}}$	2. $(16)^{\frac{3}{2}}$	3. $(125)^{\frac{1}{3}}$

Write each in radical form and then simplify. Assume all variables are positive.

4. $(27b^3c^4)^{\frac{1}{2}}$	5. $\left(\mathbf{24}\boldsymbol{x^5z^2}\right)^{\frac{1}{2}}$	6. $\left(\mathbf{24}\boldsymbol{x^5z^2}\right)^{\frac{1}{3}}$

*Simplify each based on the mathematical operation. Assume all variables are positive.

7. $\left(\mathbf{9}+\mathbf{2}\sqrt{\mathbf{5}}\right)-(\mathbf{1}+\sqrt{\mathbf{45}})$	8. $\mathbf{6}\sqrt{\mathbf{3}}(\mathbf{2}\sqrt{\mathbf{5}}+\mathbf{4}\sqrt{\mathbf{6}})$	9. $\left(\mathbf{7}-\mathbf{2}\sqrt{\mathbf{6}}\right)(\mathbf{7}+\mathbf{2}\sqrt{\mathbf{6}})$
10. $\sqrt[3]{\mathbf{3}\boldsymbol{a^2b^4}}\cdot\left(\mathbf{3}\boldsymbol{a^2b^4}\right)^{\frac{1}{3}}\cdot\sqrt[3]{\boldsymbol{ab}}$	11. $\dfrac{\sqrt{\mathbf{21}\boldsymbol{x^4y^8}}}{(\mathbf{3}\boldsymbol{x^2y^3})^{\frac{1}{2}}}$	12. $\left(\mathbf{4}+\sqrt{\mathbf{3}}\right)(-\mathbf{2}+\sqrt{\mathbf{2}})$
13. $\sqrt{\mathbf{2}\boldsymbol{x^3}}\cdot\sqrt{\mathbf{4}\boldsymbol{x^3}}$	14. $\left(\mathbf{4}\sqrt{\mathbf{5}}-\mathbf{3}\right)-(\mathbf{8}-\sqrt{\mathbf{5}})$	15. $\dfrac{-\mathbf{9}}{\sqrt{\mathbf{2}}}$
16. $\sqrt{\mathbf{56}\boldsymbol{ab^2c^{10}}}\cdot\left(\mathbf{12}\boldsymbol{b^5c^7}\right)^{\frac{1}{2}}$	17. $\left(\mathbf{4}\sqrt{\mathbf{5}}-\mathbf{3}\right)(\mathbf{8}-\sqrt{\mathbf{5}})$	18. $\dfrac{\sqrt{\mathbf{4}\boldsymbol{x^8}}}{\sqrt{\mathbf{28}\boldsymbol{x^2}}}$

Module 6: Lesson 4 – Graphing Radical Functions (Please see online book for additional examples.)

Keep in mind that a radical function is the inverse of a quadratic function.
You already should somewhat have a sense of this because you have learned that squaring and square rooting are inverse operations…they undo one another.
Therefore when you see the graph of a radical function, it will look like half of a parabola on its side.
*State the domain of each radical function and then graph.

1. Graph $y = \sqrt{x-3} + 4$

Domain: ______________ Range: ______________

2. Graph $y = -\sqrt{x} + 2$

Domain: ______________ Range: ______________

3. Graph $y = 3\sqrt{x}$

Domain: ______________ Range: ______________

4. Graph $y = 2\sqrt{x+5} - 6$

Domain: ______________ Range: ______________

5. Graph $y = \sqrt{-x} + 3$

Domain: ______________ Range: ______________

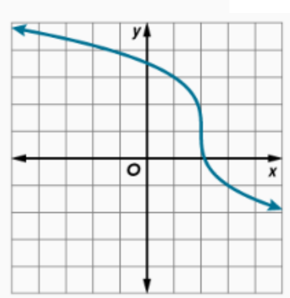
6.

Copied from Module 6: Lesson 4 - Extra Example 7.

Write a Radical Function

Write a radical function for the graph of $g(x)$.

$g(x) = \boxed{}\sqrt[3]{x - \boxed{}} + \boxed{}$



Module 6: Lesson 6 – Solving Radical Equations (Please see online book for additional examples.)

Notes:
To solve a radical equation we must get rid of the radicals wrapped around the variable.
To undo a radical, square both sides of the equation after the radical is isolated.

The Steps: 1. Isolate a radical term, 2. Square both sides, 3. Continue until all radicals are gone, 4. Solve equation, 5. Check answers in original equation.

*Solve each radical equation.

1. $(x-3)^{\frac{1}{2}} = 2$	2. $\sqrt{x+2} = 4\sqrt{x+1}$	3. $\sqrt{x^2-16} - 3 = 0$

SOLVING RADICAL EQUATIONS
* Solve each equation for the indicated unknown. Check each by graphing intersections or finding zeroes.

4. $\sqrt{x-3} + 6 = 5$	5. $\sqrt{9x^2+4} = 3x+2$	6. $\sqrt{4-2t-t^2} = t+2$	7. $\sqrt[3]{x+1} - 3 = 4$

Notes:

More on Domain Restrictions

Notes:

Definition: Domain - the set of all possible input values (usually x), which allows the function formula to work. (the x-values that are allowed to be used in a function)

Sometimes, rather than figuring out the x-values that are allowed, it is easier to figure out the x-values that are not allowed and then state that you can use all x-values except for those.

Domain restrictions can occur within even roots, like square roots, and in the denominator of fractions.

- We cannot take the square roots of negative numbers, therefore *expressions within the square root must be greater than or equal to 0.*
- We cannot divide by zero, therefore *expressions in the denominator cannot be equal to zero.*

*State the domain for each of the following functions.

Ex. 1: $f(x) = \sqrt{x+1}$	Ex. 2: $f(x) = \frac{x+4}{x-1}$	Ex. 3: $f(x) = x^2 - 3x + 4$

Practice

* State the domain for each of the following functions.

1. $f(x) = \sqrt{5x - 13}$	2. $f(x) = \frac{x+1}{(3x-4)(x+2)}$	3. $f(x) = 6x^3 + 6x - 1$
4. $f(x) = \sqrt{5x + 1}$	5. $f(x) = \frac{x-9}{(x-1)(2x+7)}$	6. $f(x) = x^2 - 3x + 4$