MODULES 7 & 8 – EXPONENTIAL AND LOGARITHMIC FUNCTIONS

1. No Calculators
2. Fill in values that you already know (No guessing)
3. Figure out more values by multiplying dividing or looking for patterns.
4. Answer the questions at the bottom of this paper.

Write the answers to each of the following. You should not need to use a calculator for this.

$1^2 =$	1
$2^2 =$	4
$3^2 =$	9
$4^2 =$	16
$5^2 =$	25
$6^2 =$	36
$7^2 =$	49
$8^2 =$	64
$9^2 =$	81
$10^2 =$	100
$11^2 =$	121
$12^2 =$	144

Fill in the missing cells of the table by following the pattern.

2^5	32
2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{4}$
2^{-3}	$\frac{1}{8}$
2^{-4}	$\frac{1}{16}$
2^{-5}	$\frac{1}{32}$

Fill in the missing cells of the table by following the pattern.

$\left(\frac{1}{5}\right)^5$	$\frac{1}{3125}$
$\left(\frac{1}{5}\right)^4$	$\frac{1}{625}$
$\left(\frac{1}{5}\right)^3$	$\frac{1}{125}$
$\left(\frac{1}{5}\right)^2$	$\frac{1}{25}$
$\left(\frac{1}{5}\right)^1$	$\frac{1}{5}$
$\left(\frac{1}{5}\right)^0$	1
$\left(\frac{1}{5}\right)^{-1}$	5
$\left(\frac{1}{5}\right)^{-2}$	25
$\left(\frac{1}{5}\right)^{-3}$	125
$\left(\frac{1}{5}\right)^{-4}$	625
$\left(\frac{1}{5}\right)^{-5}$	3125

Notice that when exponents are zero or negative, the original meaning of an exponent has to change. For example, $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$, but what about 2^0 or 2^{-3}. Explain why 2^0 or 2^{-3} have the values described in your work above.

Evaluate each expression given in exponential form.

Ex: $5^{-2} = \frac{1}{25}$	1. $3^0 =$ 1	2. $5^3 =$ 125	3. $3^{-2} =$ $\frac{1}{9}$
4. $2^{-3} =$ $\frac{1}{8}$	5. $5^{-2} =$ $\frac{1}{25}$	6. $3^4 =$ 81	7. $4^{-3} =$ $\frac{1}{64}$

*Solve each using Guess & Check. Round answers to the nearest hundredth.

8. $10^x = 14517$ $x \approx 4.16$	9. $4 \cdot 2^x = 1000$ $x \approx 7.97$	10. $2^x = 8^{2x+1}$ $x = -.6$

Simplify each expression. Leave answers with only positive exponents.

11. $3^8 \cdot 3^4$ 3^{12}	12. $(2^5)^3$ 2^{15}	13. 5^{-2} $\frac{1}{25}$	14. $\frac{4^7}{4^2}$ 4^5	15. 2^{-5} $\frac{1}{32}$
16. $5^{-3} \cdot 5^5$ 5^2	17. $(3^2)^4$ 3^8	18. $\frac{6^{11}}{6^{13}}$ $\frac{1}{6^2}$	19. $2^{-8} \cdot 2^4$ $\frac{1}{2^4}$	20. $\frac{5^{10}}{5^7}$ 5^3
21. $4^2 \cdot 2^3$ 2^7	22. $\frac{27^2}{3^5}$ 3	23. $5^{13} \cdot 5^{11}$ 5^{24}	24. $(5^3)^{13}$ 5^{39}	25. $\frac{5^3}{5^{13}}$ $\frac{1}{5^{10}}$
26. 5^{-3} $\frac{1}{125}$	27. $(5)^5 \cdot (25)^2$ 5^9	28. $\frac{(25)^2}{(5)^5}$ $\frac{1}{5}$	29. $\frac{(5)^5}{(25)^2}$ 5	30. $\frac{(25)^5}{(5)^2}$ 5^8
31. $7^2 \cdot 7^7$ 7^9	32. $\frac{7^2}{7^7}$ $\frac{1}{7^5}$	33. $7^7 \div 7^2$ 7^5	34. $(7^2)^7$ 7^{14}	35. $(7^7)^2$ 7^{14}
36. $(7^2)^{\frac{1}{2}}$ 7	37. $9^{\sqrt{3}} \cdot 27^{\sqrt{3}}$ $3^{2\sqrt{3}} \cdot 3^{3\sqrt{3}}$ $3^{5\sqrt{3}}$			

Module 7: Lesson 2 Solving Exponential Equations

Solve each equation using a "like base" strategy. Check your solutions.

38. $3^x = 3^8$ $x = 8$	39. $2^x = 8^{2x+1}$ $x = \frac{-3}{5}$	40. Solve $2^{x+4} = 4^x$ $x = 4$
41. Solve $3^{x+1} = 3^{2x}$ $x = 1$	42. Solve $12^x = 12^5$ $x = 5$	43. Challenge: $3^{x+1} = 5^x$

44. Copied from Module 7: Lesson 2 - Extra Example 1

Solve Exponential Equations Algebraically

Solve $2^{9x} = 64^{3x+6}$.

$x = \boxed{-4}$

Scientific Notation

Here are some examples of numbers in scientific notation.	Here is what certain values would look like w/o scientific notation.
Approximate distance from Earth to the Sun: $1.5 \cdot 10^8$ km Length of a red blood cell: $8.0 \cdot 10^{-6}$ meters Approximate World Population: $7.2 \cdot 10^9$ people	Mass of Earth: 13 170 000 000 000 000 000 000 000 lbs. Approximate distance from Earth to the Sun: 93,000,000 miles Mass of an electron: 0.000000000000000000000000000000091093822 kg

When comparing the two sets of examples above, why do you think Scientific Notation is used outside of a school building?

Review properties of exponents if needed.

To write a number in Scientific Notation:
Scientific Notation shows the product of two numbers.
The first number in scientific notation must be between 1 and 10.
The second number is the appropriate power of 10 that produces an equivalent value to the number not in scientific notation.

Write out each number shown in scientific notation.	Write each of the following values in scientific notation.
1. Approximate distance from Earth to the Sun: $1.5 \cdot 10^8$ km 150000000 km	4. Mass of Earth: 13 170 000 000 000 000 000 000 000 lbs. 1.317×10^{25} lbs
2. Length of a red blood cell: $8.0 \cdot 10^{-6}$ meters .000008 meters	5. Approximate distance from Earth to the Sun: 93,000,000 miles 9.3×10^7 miles
3. Approximate World Population: $7.2 \cdot 10^9$ people 7200000000 people	6. Mass of an electron: 0.000000000000000000000000000000091093822 kg $9.1093822 \times 10^{-31}$

*Evaluate each without a calculator, and leave your answer in scientific notation. Check answers using your calculators.

7. $(3.2 \cdot 10^8)(2 \cdot 10^5)$

$6.4 \cdot 10^{13}$

8. $(6 \cdot 10^8)(3 \cdot 10^{-2})$

$18 \cdot 10^6$

1.8×10^7

9. $(14 \cdot 10^{-6})(5 \cdot 10^{-5})$

70×10^{-11}

7×10^{-10}

10. $\frac{(9 \cdot 10^7)}{(3 \cdot 10^4)}$

3×10^3

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9.3×10^7 miles

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70×10^{-11}

7×10^{-10}

10. $\frac{(9 \cdot 10^7)}{(3 \cdot 10^4)}$

3×10^3

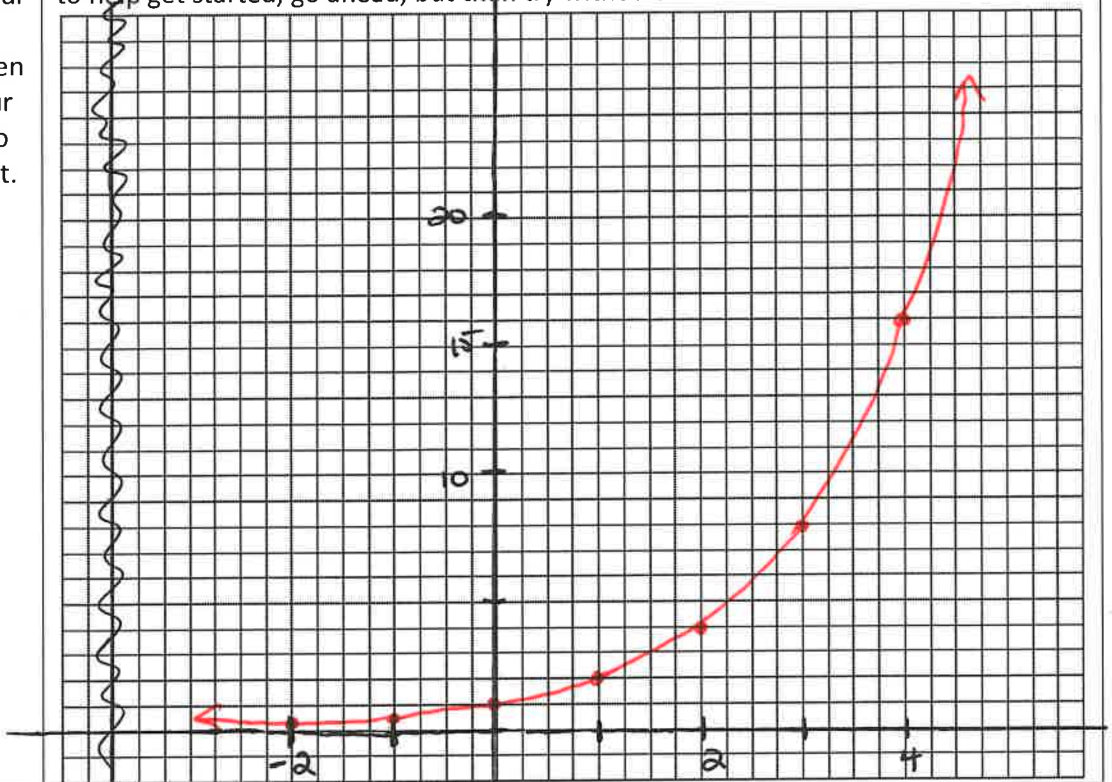
Module 7: Lesson 1 – Graphing Exponential Functions

1. a. Fill in the table for the following exponential function. Try not to use your calculator and instead of decimals, use fractions when necessary. If you need your calculator to get started, go ahead, but then try without.

$y = 2^x$

x	y
−2	1/4
−1	1/2
0	1
1	2
2	4
3	8
4	16

1. b. Set up a scale for your graph by using the information that you filled in for the table, and then graph the function $y = 2^x$. Again, if you need to use your calculator to help get started, go ahead, but then try without.



2. a. Fill in the table for the following exponential function. Try not to use your calculator and instead of decimals, use fractions when necessary. If you need your calculator to get started, go ahead, but then try without.

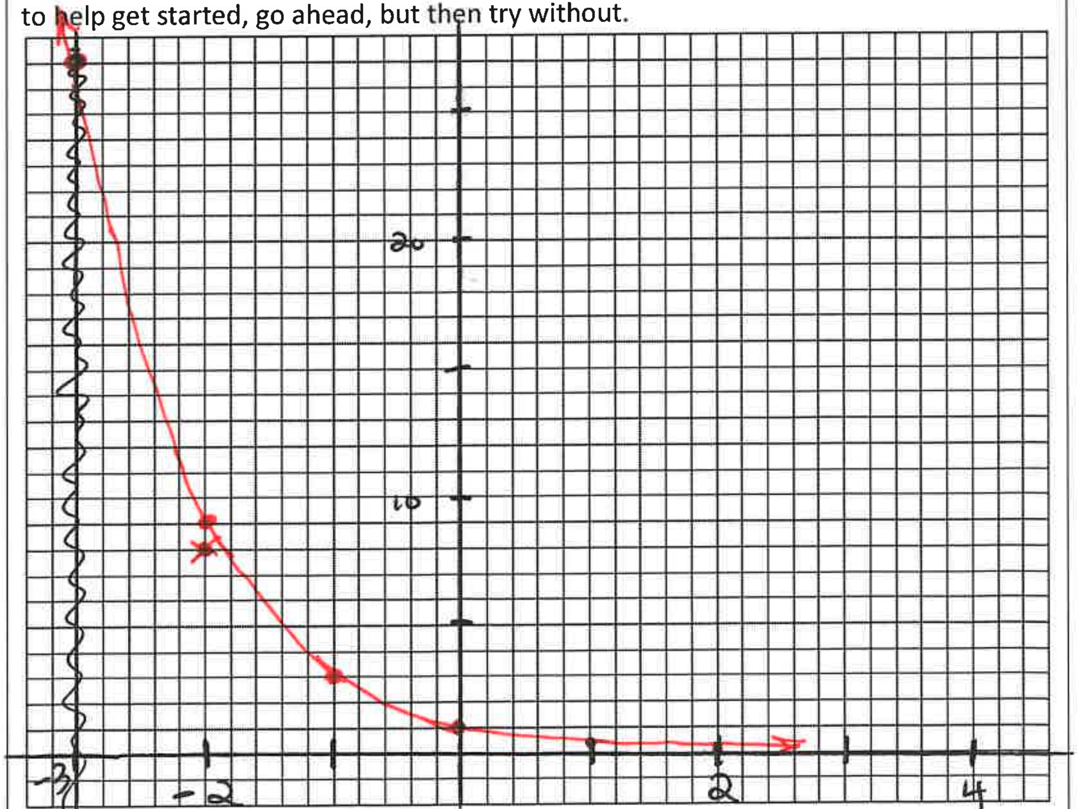
(Notice the negative exponent here.)

$y = 3^{-x}$

-3 27

x	y
−2	9
−1	3
0	1
1	1/3
2	1/9
3	1/27
4	1/81

2. b. Set up a scale for your graph by using the information that you filled in for the table, and then graph the function $y = 3^{-x}$. Again, if you need to use your calculator to help get started, go ahead, but then try without.

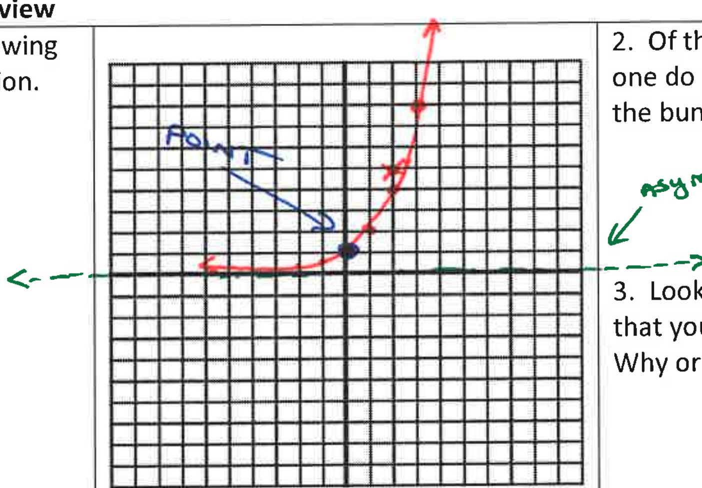


Transformation Review

1. Graph the following exponential function.

$y = 2^x$

x	y
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8



2. Of the six points that you just graphed, which one do you think is the "memorable point" of the bunch? Why?

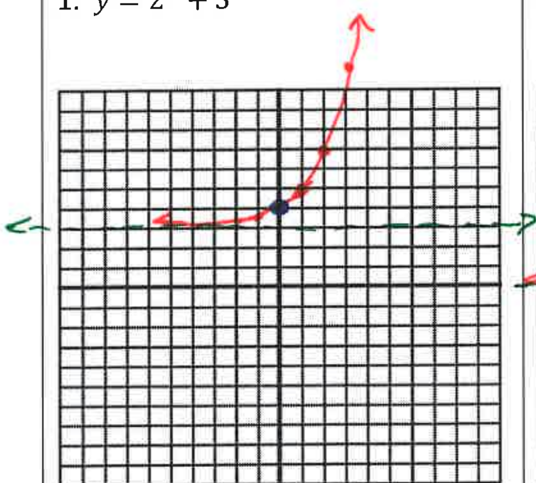
ASYMPTOTE

3. Look along the x-axis. Do you think the curve that you graphed will ever touch the x-axis? Why or why not?

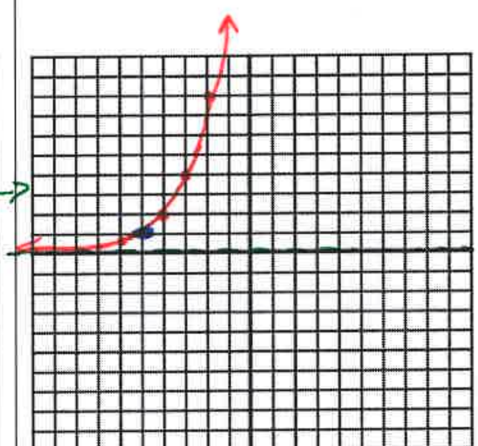
Definition: Asymptote – a reference line that a curve will approach, but never quite reach

* Use what you know about transformations, the "memorable point", and the asymptote to graph each. If you need to use your graphing calculator for the first two questions, please do so. Your goal is to not have to use your calculators.

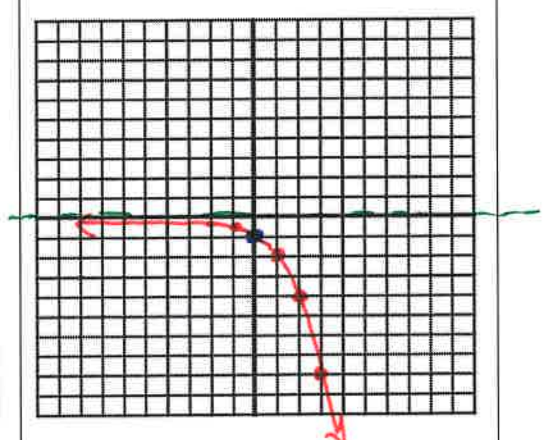
1. $y = 2^x + 3$



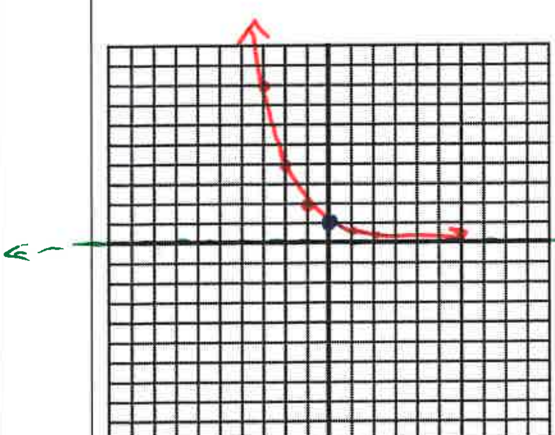
2. $y = 2^{x+5}$



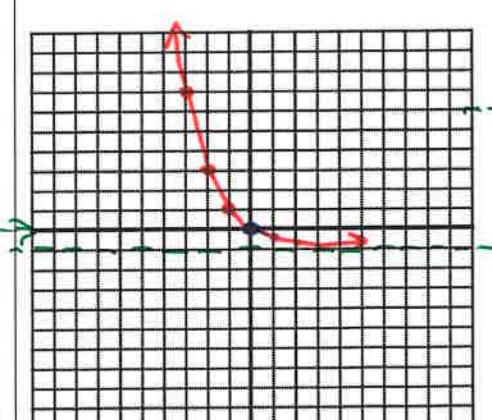
3. $y = -2^x$



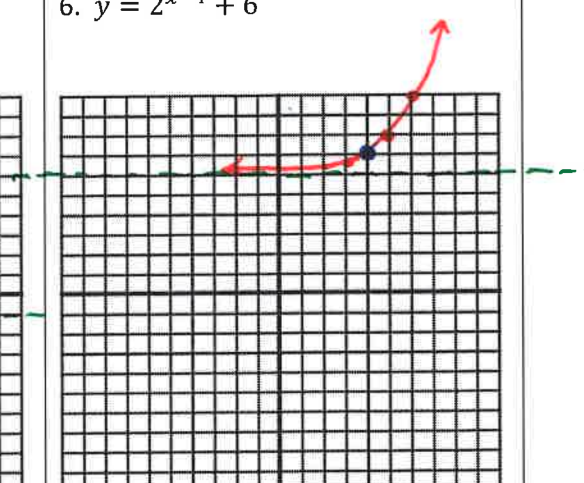
4. $y = 2^{-x}$



5. $y = 2^{-x} - 1$



6. $y = 2^{x-4} + 6$



* Consider the function $y = f(x)$.

Example: If I wished to move this function two up, I would write this function as $y = f(x) + 2$.	Example: If I wished to move this function seven to the left, I would write this function $y = f(x + 7)$.

Write the adjusted function for each of the following:

7. Reflect the function over the x-axis. $y = -f(x)$	8. Move the function two to the right and three down. $y = f(x-2) - 3$
9. Reflect the function across the y-axis. $y = f(-x)$	10. Reflect the function over both axes and move the function up four. $y = -f(-x) + 4$

Copied from Module 7:1 Extra Example 1.

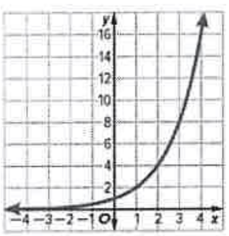
Graph Exponential Growth Functions

Consider $f(x) = 4^x$.

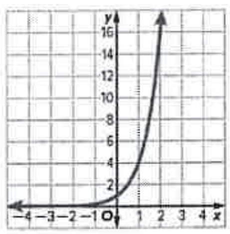
Part A

Select the graph of $f(x) = 4^x$.

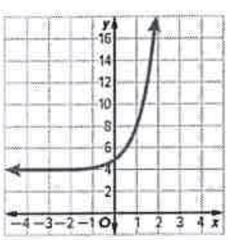
○ A



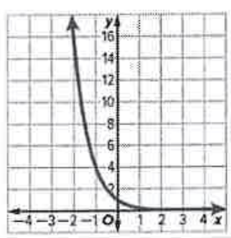
● B



○ C



○ D



Part B

Find the domain, range, y-intercept, asymptote, and end behavior.

domain: $(-\infty, \infty)$

range: $(0, \infty)$

y-intercept: $(0,1)$

asymptote: $y=0$

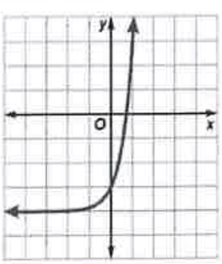
end behavior: as $x \to -\infty, f(x) \to$ 0 and as $x \to \infty, f(x) \to$ ∞

Copied from Module 7:1 Extra Example 3.

Analyze Graphs of Exponential Functions

Identify the value of k and write a function for each graph as it relates to $f(x) = 7^x$.

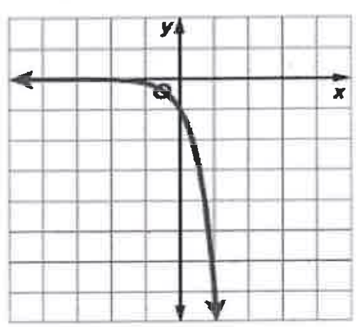
a. $g(x) = f(x) + k$



$k =$ -4

$g(x) =$ $7^x - 4$

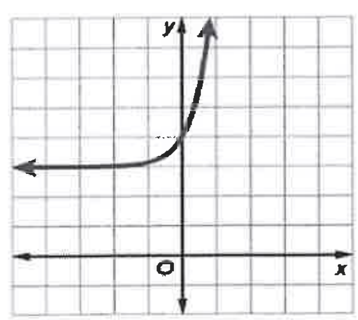
b. $h(x) = k \cdot f(x)$



$k =$ -1

$h(x) =$ $-1 \cdot 7^x$

c. $j(x) = f(x) + k$



$k =$ 3

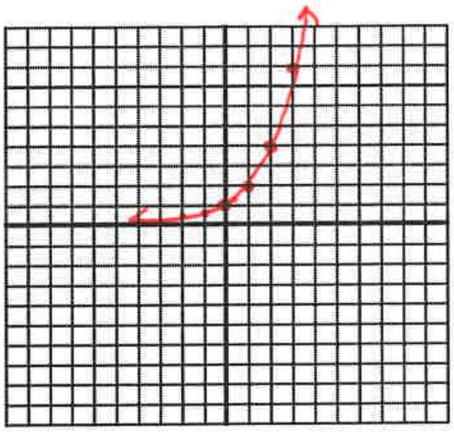
$j(x) =$ $7^x + 3$

Module 7: Lesson 2 Solving Exponential Equations (Applications of Exponents and Logarithms)

Review – Graph the following exponential functions.

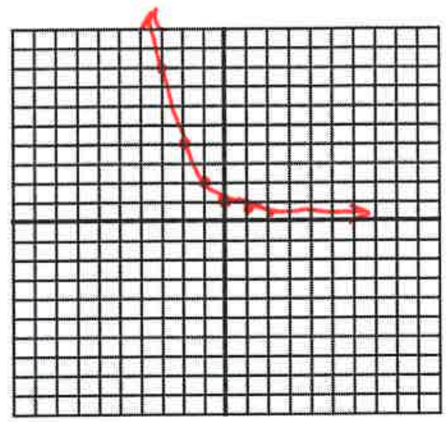
1. $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$1/2$
0	1
1	2
2	4
3	8



2. $y = 2^{-x}$

x	y
-3	8
-2	4
-1	2
0	1
1	$1/2$
2	$1/4$



As you look at both graphs, what point is consistent in both situations? Why is this?

Again as you look at both graphs, both seem to be getting closer and closer to a reference line called an asymptote. How would you describe the location of this reference line on the graphs above?

You should see a connection with the two graphs/equations and what you know about transformations. Write a short explanation regarding this connection.

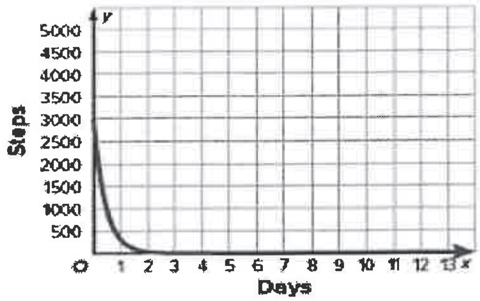
When looking at the equation for an exponential function, how can you tell that it is describing decay?

Copied from Module 7:1 Extra Example 4.

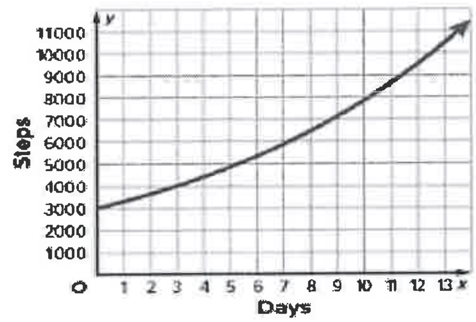
Part A

Select a graph to represent the number of steps Addison takes.

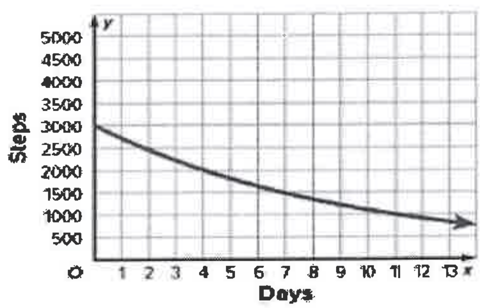
○ A



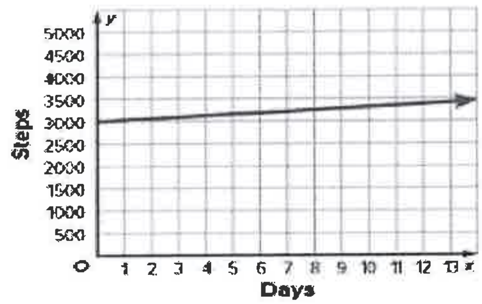
● B



○ C



○ D



Use Exponential Growth Functions

FITNESS Addison is participating in a fitness challenge. Each day, for 2 weeks, she wants to increase the number of steps she takes by 10%. If Addison takes 3000 steps the first day, use a graphing calculator to graph and interpret a function representing this situation.

Part B

Estimate the number of steps she will take on the last day of the challenge.

ABOUT

11,400 steps

Look these up:

Boiling Point of H_2O	Freezing Point of H_2O
100 Celsius or 212 Fahrenheit	0 Celsius or 32 Fahrenheit

Activity: Observing Exponential Decay

Objectives:

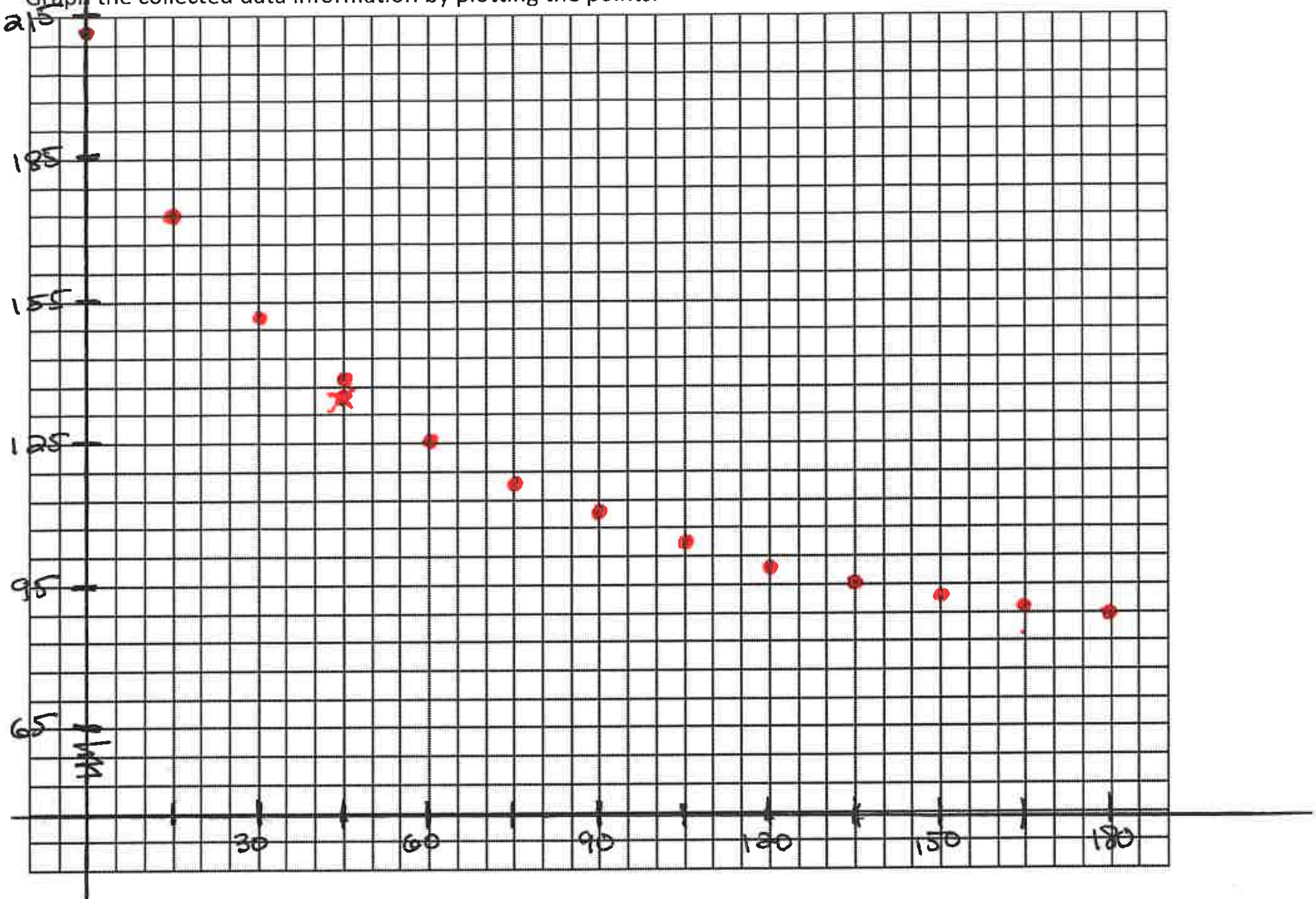
a. Observe exponential decay in real-time
b. Make predictions about decay over time
c. Identify that functions can be represented as tables, graphs, and equations
d. Calculate a regression function with given data
e. Observe and predict idea of a limit

Module 7: Lesson 5 – Modelling Data

Fill in the table below based on the measurements as the boiling water cools. **Room Temperature:** 75

Time (t)	0	15	30	45	60	75	90	105	120	135	150	165	180
Temp. (T)	212	173	152	138	125.5	117	111	104	99.5	95.5	93	90.5	89

Graph the collected data information by plotting the points.



How would you describe the shape or behavior of this graph?	We have discussed a reference line called an asymptote. Your graph is approaching an invisible horizontal line. How would you describe the location of this reference line?
EXPONENTIAL DECAY	HORIZONTAL ASYMPTOTE IS ROOM TEMP $y = 75$

https://www.youtube.com/watch?[illegible]

On your calculators, use STAT -> EDIT to input the data from the table into lists L1 and L2.
Talk with your teacher on how to adjust your data for the horizontal asymptote.

Time (t)	0	15	30	45	60	75	90	105	120	135	150	165	180
Temp. (T)													
Fool the Calculator	137	98	77	63	50.5	42	36	29	24.5	20.5	18	15.5	14

On your calculators, use STAT -> EDIT to input the data from the table.
Now use STAT -> CALC -> ExpReg to calculate the equation for the best equation that fits this data.

Write your equation here:	Now adjust this equation to fit the actual data from the experiment.
$y = 114.46(.98766)^x$	$y = 114.46(.98766)^x + 75$

1. Increase 30 by 70%. 51	2. Decrease 250 by 80%. 50
3. 450 increased by 35%. 607.5	4. 145 decreased by 67%. 47.85
5. Increase your answer from #1 by 20%. 61.2	6. Increase your answer from #2 by 70%. 85

<u>The Multiplier</u> -> The "High School" method of increasing or decreasing by various percentages. Many applications for exponential growth/decay.

3. If $10,000 is deposited in a savings account earning 4% interest every year, how much money is in the account after 10 years? $10000(1+.04)^{10}$ $14 802	4. A $17,000 car depreciates in value by 15% each year. How much will this car be worth in six years? $17000(1-.15)^{6}$ $1 6412

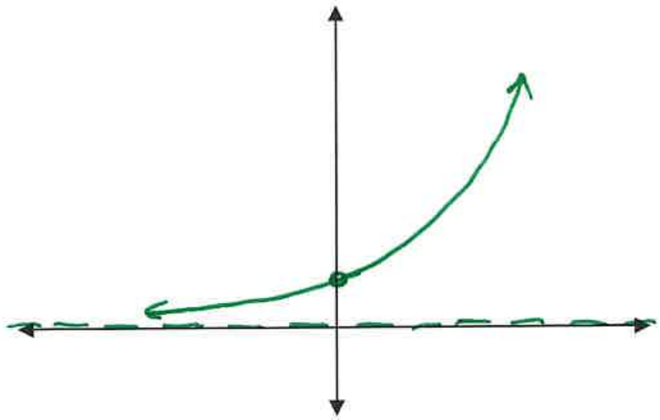
Identify whether each exponential function is growth or decay. Also, discuss the details of each function.

9. $y = 5(1.25)^x$ G	10. $A = 1000(1.08)^t$ G	11. $f(x) = 5(.92)^x$ D
12. $g(x) = 54 \cdot \frac{1}{2}^x$ D	13. $y = 10(2)^{-x}$ D	14. $A = 12\left(\frac{1}{3}\right)^x$ D

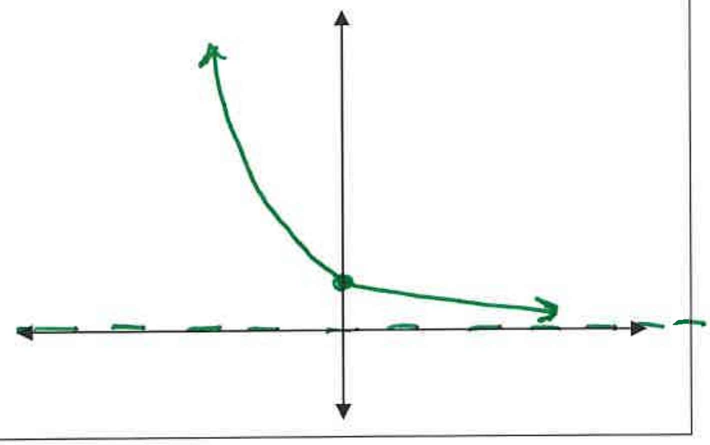
12. $b(x) = 40(3.8)^x$ G	13. $z(x) = 0.4^x$ D	14. $k(x) = 500(.99)^x$ D	15. $f(x) = 17(4)^x$ G

7. Frank invests $12,000 in an account that earns 7.5% interest compounded annually. What is the total amount of his investment after 3 years? $12000(1+.075)^3$ $14908	5. Compare the ending amounts of a $5000 investment at 18% annual interest for 10 years when:	
	a. Compounded Yearly $5000(1+.18)^{10}$ $26169	b. Compounded Weekly $5000\left(1+\frac{.18}{52}\right)^{10 \cdot 52}$ $30154
	c. Compounded Daily $5000\left(1+\frac{.18}{365}\right)^{10 \cdot 365}$ $30235	d. Compound every second

6. Draw a rough sketch of exponential growth.
(Keep in mind the "memorable point" and the horizontal asymptote.)



6. Draw a rough sketch of exponential decay.
(Keep in mind the "memorable point" and the horizontal asymptote.)



8. Discuss the information in this table.

AGE	BEN INVESTS:		ARTHUR INVESTS:	
19	2,000	2,240	0	0
20	2,000	4,749	0	0
21	2,000	7,558	0	0
22	2,000	10,706	0	0
23	2,000	14,230	0	0
24	2,000	18,178	0	0
25	2,000	22,599	0	0
26	2,000	27,551	0	0
27	0	30,857	2,000	2,240
28	0	34,560	2,000	4,749
29	0	38,708	2,000	7,558
30	0	43,352	2,000	10,706
31	0	48,554	2,000	14,230
32	0	54,381	2,000	18,178
33	0	60,907	2,000	22,599
34	0	68,216	2,000	27,551
35	0	76,802	2,000	33,097
36	0	85,570	2,000	39,309
37	0	95,383	2,000	46,266
38	0	107,339	2,000	54,058
39	0	120,220	2,000	62,785
40	0	134,646	2,000	72,559
41	0	150,804	2,000	83,506
42	0	168,900	2,000	95,767
43	0	189,168	2,000	109,499
44	0	211,869	2,000	124,879
45	0	237,293	2,000	142,104
46	0	265,768	2,000	161,396
47	0	297,660	2,000	183,004
48	0	333,379	2,000	207,204
49	0	373,385	2,000	234,308
50	0	418,191	2,000	264,665
51	0	468,374	2,000	298,665
52	0	524,579	2,000	336,745
53	0	587,528	2,000	379,394
54	0	658,032	2,000	427,161
55	0	736,995	2,000	480,660
56	0	825,435	2,000	540,579
57	0	924,487	2,000	607,688
58	0	1,035,425	2,000	682,851
59	0	1,159,676	2,000	767,033
60	0	1,298,837	2,000	861,317
61	0	1,454,698	2,000	966,915
62	0	1,629,261	2,000	1,085,185
63	0	1,824,773	2,000	1,217,647
64	0	2,043,746	2,000	1,366,005
65	0	$2,288,996	2,000	$1,532,166

9. What are the implications represented in the table to the left?

IN CLASS DISCUSSION

10. Use an investment application online to explore different investment possibilities. Write down some of these options.

Copied from Module 7: Lesson 5 – Example 1

Examine Scatter Plots

Use a graphing calculator to make a scatter plot of the data in the table. Then determine whether the data are best modeled by a *linear*, *quadratic*, or *exponential* function.

x	−5	−4	−3	−2	−1	0	1	2	3	4
y	−3	−2.7	−2.4	−2	−1	0	2.8	8	17.2	31

Copied from Module 7: Lesson 5 – Example 2

Model Data by Using Technology

IN CLASS DISCUSSION

COFFEE Forty years after opening their first shop, a popular coffee company grew to over 17,000 locations worldwide. The table shows the number of shops in the years since the company began. Use the data to determine a best-fit model. Then, use the model to make predictions.

Coffee Shop Growth

Years Since Opening	Shops	Years Since Opening	Shops
0	1	28	2498
16	17	30	4709
18	55	32	7225
20	116	34	10,241
22	272	36	15,011
24	677	38	16,635
26	1412	40	17,003

Copied from Module 7: Lesson 5 – Extra Example 2

Model Data by Using Technology

WORLD POPULATION It is estimated that the world population reached 1 billion around 1804. Since then, it has been increasing at a dramatic pace. The table shows the estimated world population in billions since 1900. Use a calculator to determine the model of best fit. Then, use the model to predict.

Years Since 1900	Population (billions)
0	1.656
50	2.558
60	3.043
70	3.713
80	4.451
90	5.289
100	6.089
110	6.892

Part B
Use the model to predict.

Use the original data to estimate the world population in the given years.

a. 1930: ☐ **b.** 1995: ☐ **c.** 2020: ☐

Use the model of best fit to predict the world population in the given year

a. 1930: ☐ **b.** 1995: ☐ **c.** 2020: ☐

*Solve each using Guess & Check on your calculators. Round answers to the nearest hundredth.

1. $10^x = 14517$ $x \approx 4.16$	2. $2^x = 1000$ $x \approx 9.97$	3. $5.3^x = 123456$ $x \approx 7.03$

These questions have "nice" solutions, so do this set without a calculator.

2. $2^x = 8$ $x = 3$	2. $5^x = 25$ $x = 2$	2. $2^x = \frac{1}{8}$ $x = -3$	2. $3^x = 81$ $x = 4$
2. $4^x = 64$ $x = 3$	2. $2^x = 64$ $x = 6$	2. $3^x = \frac{1}{9}$ $x = -2$	2. $5^x = 125$ $x = 3$

MODULE 8: Lesson 1 – Logarithms and Logarithmic Functions

The word logarithm originates from two Greek words: *Logos meaning reason or* *logic* *Arithmus meaning* *number* So the word logarithm basically means a logical number.	Example: The logarithm base 10 of 1000 looks like this: $log_{10}1000$ Which means… "To what number do you have to raise 10 to get 1000?" The answer is 3 because $10^3 = 1000$. Therefore, $log_{10}1000 = 3$.

Try these. Evaluate each. The first answer is given as an example.

1. $log_2 8 = 3$	2. $log_5 25$ $= 2$	3. $log_2 32$ $= 5$	4. $log_4 64$ $= 3$
5. $log_4 4$ $= 1$	6. $log_4 1$ $= 0$	7. $log_2 2$ $= 1$	8. $log_2 1$ $= 0$
9. $log_2\left(\frac{1}{8}\right)$ $= -3$	10. $log_3 81$ $= 4$	11. $log_3\left(\frac{1}{27}\right)$ $= -3$	12. $log_5\left(\frac{1}{5}\right)$ $= -1$

* Introduction to Solving Logarithmic Equations - Solve each for x.

13. $log_2 8 = x$ $x = 3$	14. $log_5 5 = x$ $x = 1$	15. $log_{10}\left(\frac{1}{100}\right) = x$ $x = -2$	16. $3^x = 3^7$ $x = 7$
17. $7^{x+3} = 7^{2x+1}$ $x = 2$	18. $(2)^{x+4} = (4)^x$ $x = 4$	19. $log_x 49 = 2$ $x = 7$	20. $log_3 x = 4$ $x = 3^4$ or 81

Solve each for x.

9. $log_x 4 = \frac{1}{2}$ $x = 16$	10. $log_{10} x = log_{10} 6$ $x = 6$	11. $log_7(x+1) = log_7 7$ $x = 6$	12. $log_3 2x = 2$ $x = 4.5$
13. $log_3(3^2) = x$ $x = 2$	14. $log_x 25 = \frac{2}{3}$ $x = 125$	15. $log_{27} 9 = x$ $x = \frac{2}{3}$	

Find the value of v in each equation.

19. $v = log_{10} 1000$ $v = 3$	20. $v = log_{15} 225$ $v = 2$	21. $v = log_{12} 144$ $v = 2$	22. $8 = log_2 v$ $v = 2^8$ or 256
23. $-4 = log_4 v$ $v = 4^{-4}$ or $\frac{1}{4^4}$	24. $-3 = log_7 v$ $v = 7^{-3}$ or $\frac{1}{7^3}$	25. $-2 = log_v \frac{1}{100}$ $v = 10$	26. $log_v 729 = 6$ $v^6 = 729$ $v = 3$

* Solving Logarithmic and Exponential Equations

1. $4^x = 4^{3x-1}$ $x = \frac{1}{2}$	3. $8^2 = 4^{x+5}$ $2^6 = 2^{2x+10}$ $x = -2$	2. $log_{10}(2x) = log_{10}(x+7)$ $x = 7$
5. $3^{log_3 8} = x$ $x = 8$	4. $log_2 32 = x$ $x = 5$	6. $log_2 7x = log_2(x^2+12)$ $x^2 - 7x + 12 = 0$ $(x-3)(x-4) = 0$ $x = 3$ or 4

*Evaluate each expression.

7. $log_4 4$ 1	8. $5 \cdot log_4 4$ 5	9. $log_4 4 + log_4 4 + log_4 4 + log_4 4 + log_4 4$ 5	10. $log_4 4^5$ 5

MODULE 8: Lesson 2 – Properties of Logarithms

There is a Logarithm button on your calculator. Find it. **LOG**	The common log of 1000 can either be written $log_{10} 1000$ or log 1000. (If a base of the logarithm is not shown, it is assumed to be log base 10.) This is similar to a square root not needing to show the root index. Remember that $\sqrt{\ }$ and $\sqrt[2]{\ }$ mean the same thing.
It is a "common logarithm" or log_{10} "log base 10" and is on our calculators because our number system is base 10.	
Test it out. $log_{10} 1000 = 3$ which means that $10^3 = 1000$. Type log 1000 on your calculator. Did it work?	

QUIZ IN 2 PARTS.

I NO CALCULATOR

(PACKET PAGE 17 & QUESTIONS 2, 3, 9, 10 FROM DIE ROLL EQTNS)

II CALCULATOR

APPLICATIONS

PACKET PAGES 19, 20, 22, 24

Investigating Inverses Algebraically

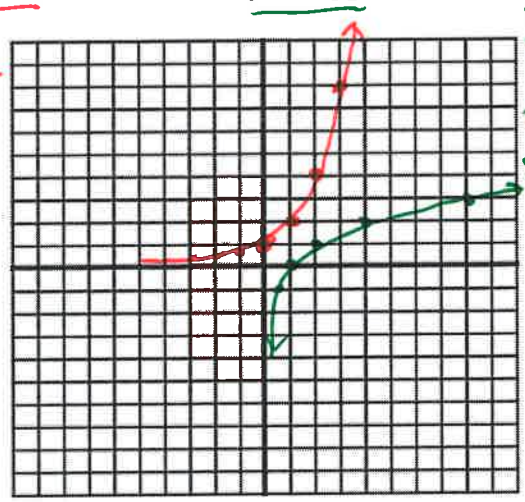
Review

1. $y = 2^x$

x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

2. $y = log_2 x$

x	y
1/8	-3
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3

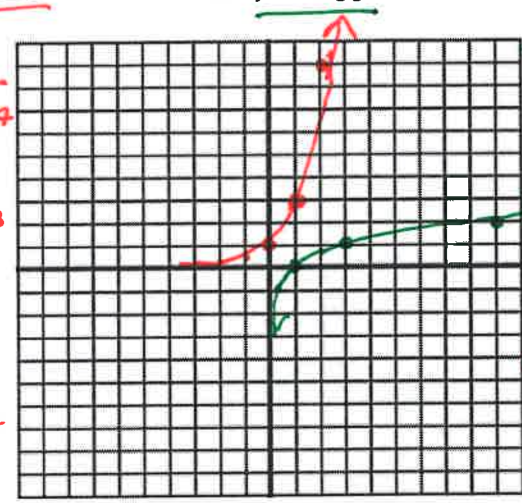


3. $y = 3^x$

x	y
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27

4. $y = log_3 x$

x	y
1/27	-3
1/9	-2
1/3	-1
1	0
3	1
9	2
27	3



Determining Inverses Algebraically.

Determine the inverse for $f(x) = \sqrt{7x - 1}$

$f^{-1}(x) = \frac{x^2+1}{7}$

Determine the inverse for $h(x) = \frac{3}{2}x - 7$

$h^{-1}(x) = \frac{2(x+7)}{3}$

Determine the inverse for $f(x) = (x - 6)^2$

$f^{-1}(x) = 6 \pm \sqrt{x}$

Determine the inverse for $g(x) = \sqrt{2x - 3}$

$g^{-1}(x) = \frac{x^2+3}{2}$

Determine the inverse for $f(x) = 2^x$

$f^{-1}(x) = \log_2 x$

Determine the inverse for $g(x) = 3^x$

$g^{-1}(x) = \log_3 x$

Determine the inverse for $h(x) = 10^x$

$h^{-1}(x) = \log x$

Determine the inverse for $h(x) = 7^x$

$h^{-1}(x) = \log_7 x$

THESE ARE THE 4 DIE ROLL EQUATIONS THAT YOU WILL BE RESPONSIBLE TO SOLVE WITHOUT A CALCULATOR.

2. $3 \cdot 11^x + 4 = 487$

$-4 \qquad -4$

$3 \cdot 11^x = 483$

$\frac{3 \cdot 11^x}{3} = \frac{483}{3}$

$11^x = 161$

$x = \log_{11} 161$

(THIS IS THE EXACT SOLUTION. COMPARE THIS w/ YOUR CALCULATED SOLUTION ON YOUR WORKSHEET.)

3. $\log(3x+1) = 2$

REMEMBER THAT THIS IS BASE 10.

$\log_{10}(3x+1) = 2$

THIS MEANS

$10^2 = 3x+1$

$-1 \qquad -1$

$3x = 99$

$x = 33$

(REMEMBER THAT LN IS LOG BASE e.)

9. $\dfrac{5 \cdot e^{x}}{5} = \dfrac{92}{5}$

$$e^{x} = \frac{92}{5}$$

CONVERT TO LOG BASE e.

$$\ln\left(\frac{92}{5}\right) = x$$

(THIS IS THE EXACT SOLUTION.)

(COMPARE THIS w/ YOUR CALCULATED SOLUTION.)

10. $\ln(2x-1) = 4$

THIS IS LOG BASE e.

$$e^{4} = 2x - 1$$

$+1 \qquad +1$

$$\frac{e^{4}+1}{2} = \frac{2x}{2}$$

$$x = \frac{e^{4}+1}{2}$$

AGAIN... AN EXACT SOLUTION.

COMPARE w/ YOUR CALCULATED SOLUTION.

Formula Bank

Compound Interest Formula (when $n = 4, 12, 365, etc.$): $A = P\left(1+\frac{r}{n}\right)^{nt}$

when $n = 1$: $A = P(1+r)^t$

when $n = \infty$: Continuously Compounded Interest: $A = Pe^{rt}$

$A = Pe^{rt}$ $A = A_o e^{rt}$ $N(t) = N_o e^{kt}$	$A = P\left(\frac{1}{2}\right)^{\frac{t}{halflife}}$	$I(dB) = 10\log\left[\frac{I}{I_o}\right]$	$M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$

Warm-Up/Review

1. Increase 45 by 36%.

   61.2

2. Decrease 167 by 13%.

   145.29

3. Decrease 56 by 20%. Continue this process for four more steps.

   1ST STEP → 44.8

   5TH STEP → 18.35

4. There is a $25 shirt that you would like to buy. The store is offering 40% off the shirt. You also have a 15% off coupon to use. Sales tax in this particular county is 6.5%. What will the cash register total be for your new shirt?

   $ 13.58

5. The value of a painting is $12,000 in 1990 and increases by 8% of its value each year. Write and evaluate an expression to estimate the paintings value in 2005.

   $ 38066

   ANSWERS CAN VARY DEPENDING ON METHOD (SHOW WORK)

6. 11 bacteria cells double every 30 minutes. How many cells will there be after:
   a. 2 hours → 176 CELLS
   b. 2.5 hours → 352 CELLS
   c. 6 hours → 45056 CELLS

7. $15,000 is placed in a savings account earning 7% interest compounded monthly. How much will be in the account if you let it sit for 15 years?

   $ 41 000 TO $ 43000 RANGE

   ANSWERS CAN VARY DEPENDING ON METHOD

8. An investor puts $5000 in an account that earns 6.5% annual interest which is compounded continuously. Find the amount that will be in the account at the end of 5 years if no deposits or withdrawals are made.

   AROUND $ 6900

my solutions reflect using e. your solutions may be slightly different if you use another method.

20

* Extra Practice: Work on these questions involving the number *e*.

9. Iodine-131 is radioactive material that follows an exponential decay model. A sample has 100 grams of iodine-131. The daily decay rate of iodine-131 is -.087. How much will be left after 9 days.

45.7 g

10. How long would it take to make $2,000 if you invested $800 at 5% interest compounded continuously?

about 18 years

11. You deposit $5000 in an account that earns 11% interest compounded continuously. How long would it take your investment to:

a. double. b. triple

6.3 yrs 10 yrs

12. You deposit $1,000 in an account that earns 3% interest compounded continuously. How much would be in the account 20 years later?

$ 1822

13. An investor wants to make his $30,000 into $50,000. What rate would he need to get on his investment if he deposits it into an account for 8 years earning continuous interest?

about 6.4% interest

14. An investment service promises to triple your money in 12 years. Assuming continuous compounding of interest, what rate of interest is needed?

about 9.2% interest

15. The population of a certain insect has a daily growth rate of .02 and follows the law of uninhibited growth. What is the population after 10 days if the initial population is 500?

about 611 insects

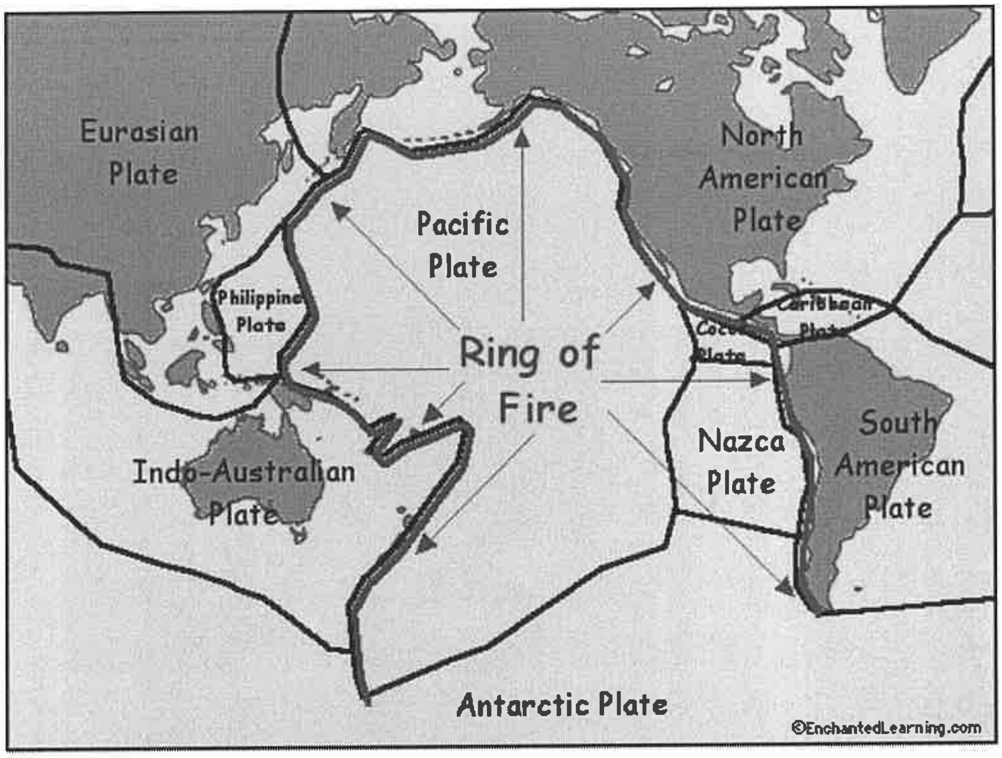
16. The population of India was estimated to be 1,056,576,000 in 2000 and 1,234,281,000 in 2010. Assume that this population growth is exponential.

a. $k \approx .0155$ or about 1.55%

a. Use the exponential growth function, $P(t) = P_o e^{kt}$, to find the value of k.
b. Estimate the population in India this year, rounded to the nearest hundred thousand.
c. Use the function you wrote in part a to estimate the year in which the population will reach 2 billion.

b. 1.51 billion people estimated

c. about 2041



1. Jammu and Kashmir is the northernmost state of India. It is situated mostly in the Himalayan mountains. In 2005, an earthquake was reported in this region with an energy level of 1.12×10^{22} ergs. What was the Richter Scale reading of this earthquake? 6.8	2. The "infamous" San Francisco earthquake of 1906 measured 8.0 on the Richter Scale. What was the energy level of this particular earthquake? $10^{23.8}$ ERGS
3. From December 1811 to early 1812, a series of earthquakes shook the Mississippi Valley near New Madrid, Missouri. One of the earthquakes released about 2.5×10^{24} ergs of energy. Find the earthquake's magnitude on the Richter Scale. Round your answer to the nearest tenth. 8.4	4. An earthquake was measured to have a Richter Scale reading of 7.2. What was the energy level of this particular earthquake? $10^{21.8}$ ERGS

3. Plutonium-238 has a half-life of 88 years.

a. Determine the decay rate, r, of Plutonium-238.	b. How long would it take a sample of Plutonium-238 to decay to 75% of its original amount .	c. If a 1200 gram sample of Plutonium-238 decayed for 300 years, how much of the Plutonium-238 would be around after this time period?
$r \approx -.00788$	ABOUT 36.5 YEARS	ABOUT 113 g

4. Use decay constant, r, for Carbon-14 dating (r = -.000121) to help answer the following questions.

a. If an original sample of C-14 weighed 1,200 g, how much Carbon-14 would remain after 2000 years?	b. A certain object, that was found in a dig, contained 50% of its original Carbon-14. Approximately how old was the object?	c. Another object contained 30% of its original Carbon-14. Approximately how old is this object?
942 g	5730 YEARS	9950 YEARS
d. What is the half-life of Carbon-14? (Hint: Think time)	e. How much of a 200g sample of Carbon-14 would still be around after 1000 years?	f. An ancient artifact is found. Scientists determine that the artifact has 40% of its original Carbon-14 remaining. Approximately how old is the artifact?
5730 YEARS	177 g	7573 YEARS OLD