

INTRODUCTION TO POLYNOMIALS

Patterns in Polynomials

* For each equation, fill in the table and then determine the pattern of the function values (aka y-values). Continue this process of finding patterns until you notice a constant pattern, i.e., a pattern with the same number repeated. You may need to work through this process two or three or more times.

1. $y = x^2$

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y														

2. $y = 5x - 11$

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y														

3. $y = 2x^3 - 4x^2 + 5x - 7$

Use your graphing calculator to help you fill in this table.

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y														

Classifying Polynomials (see also Mod 4: Lesson 1)

By Degree (the largest exponent)

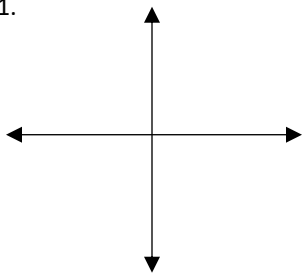
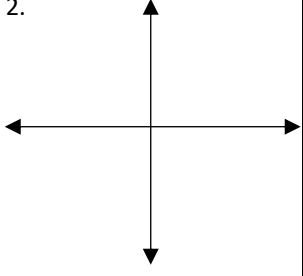
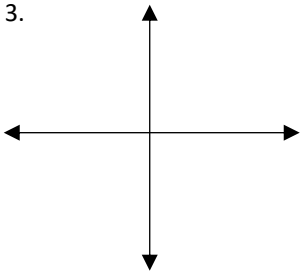
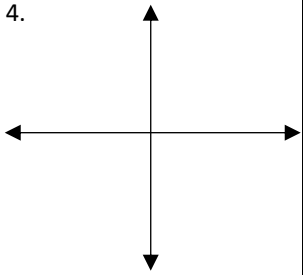
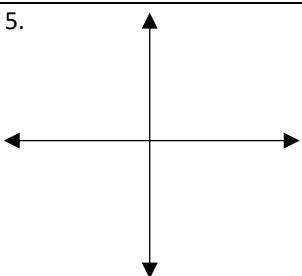
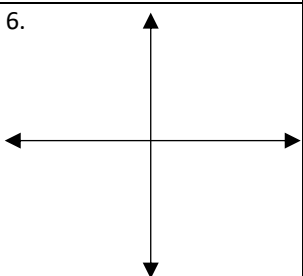
Degree	Classification
0	constant
1	linear or first degree
2	quadratic or second degree
3	cubic or third degree
4	fourth degree
5	5 th degree

By Number of Terms

Number of Terms	Classification
1	Monomial
2	Binomial
3	Trinomial
4	4 terms
5	5 terms

Rough Sketches

* Use your calculator to graph each function. Sketch the shape of each graph beside each function. Also, classify the polynomial by degree and number of terms as well as stating the leading coefficient.

1.a. $y = x^2 + x - 2$	1. 	2.a. $y = 2x - 5$	2. 
3.a. $y = -2x^3 + 4x^2 + x - 2$	3. 	4.a. $y = -x^4 + 5x^3 + 5x^2 - x - 6$	4. 
5.a. $y = x^4 + 2x^3 - 5x^2 - 6x$	5. 	6.a. $y = -3x^2 + x^3 - 3x$	6. 

End Behavior (see also Mod 4: Lesson 1)

The **end behavior** of a polynomial **function** is the **behavior** of the graph of $f(x)$ as x approaches positive infinity or negative infinity. The degree and the leading coefficient of a polynomial **function** determines the **end behavior** of the graph.

To talk about end behavior, think about the direction the arrows would be pointing if you graphed each by hand.

* Use your sketches to help fill in the table below. Think about what may cause turns and end behavior.

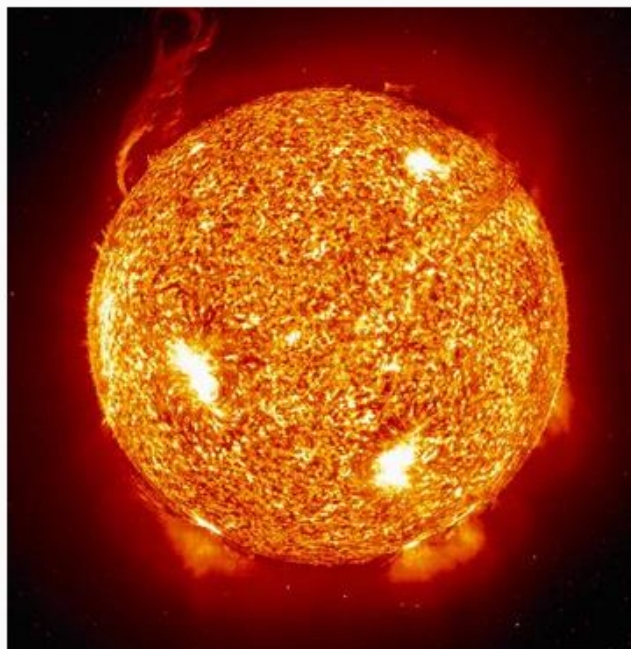
Equation	Degree	# of Turns	End Behavior
1.b. $y = x^2 + x - 2$			
2.b. $y = 2x - 5$			
3.b. $y = -2x^3 + 4x^2 + x - 2$			
4.b. $y = -x^4 + 5x^3 + 5x^2 - x - 6$			
5.b. $y = x^4 + 2x^3 - 5x^2 - 6x$			
6.b. $y = -3x^2 + x^3 - 3x$			

Copied from Module 4:1 Example 4.



McGraw-Hill Education

SUN The density of the Sun, in grams per centimeter cubed, expressed as a percent of the distance from the core of the Sun to its surface can be modeled by the function $f(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$, where x represents the percent as a decimal. At the core $x = 0$, and at the surface $x = 1$.



Operations with Polynomials (see also Mod 4: Lesson 3)

* Write each sum or difference as a polynomial in standard form. Also, classify the polynomial by degree and number of terms as well as stating the leading coefficient.

1. $(x^3 + x^2 + x + 1) + (2x^3 + 3x^2 + x + 3)$	2. $(1 - 5x + x^3) - (2x^4 + 5x^3 - 10x^2)$

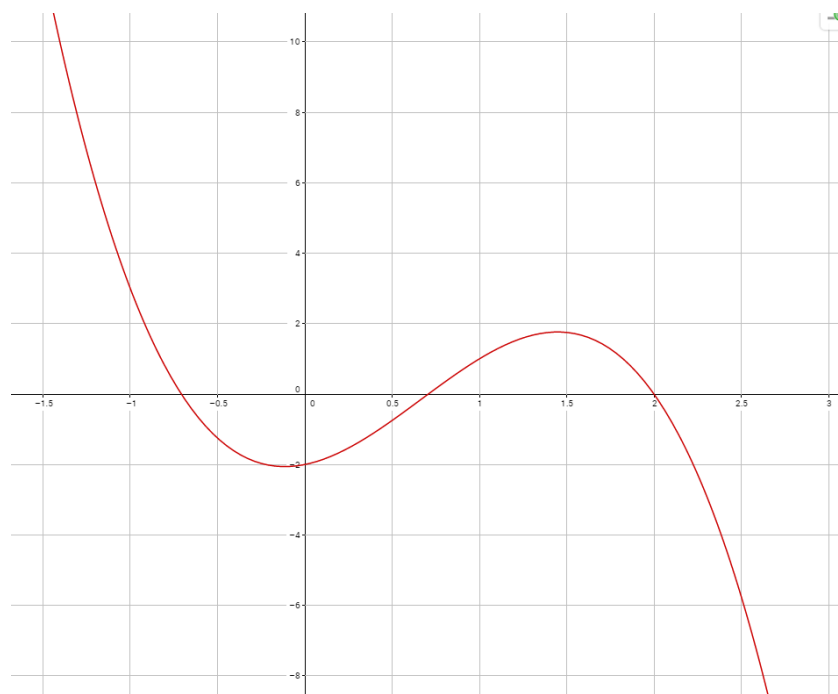
* Multiply each. Also, classify the polynomial by degree and number of terms as well as stating the leading coefficient.

3. $(x + 2)(x - 7)$	4. $7x(x^2 - 3x + 1)$	5. $(x + 2)(x^2 + 5)$
6. $(x^2 - 2)(x^3 - x)$	7. $(x + 2)(x^2 - 3x + 1)$	8. $(x^2 - 3x + 1)(2x^2 + x - 6)$

Extrema...Local Minima/Maxima (see also Mod 4: Lesson 2)

* Answer each question regarding the characteristics of each function. Approximate all answers to the nearest hundredth.

1. $f(x) = -2x^3 + 4x^2 + x - 2$



a. Approximate coordinates of local maximums

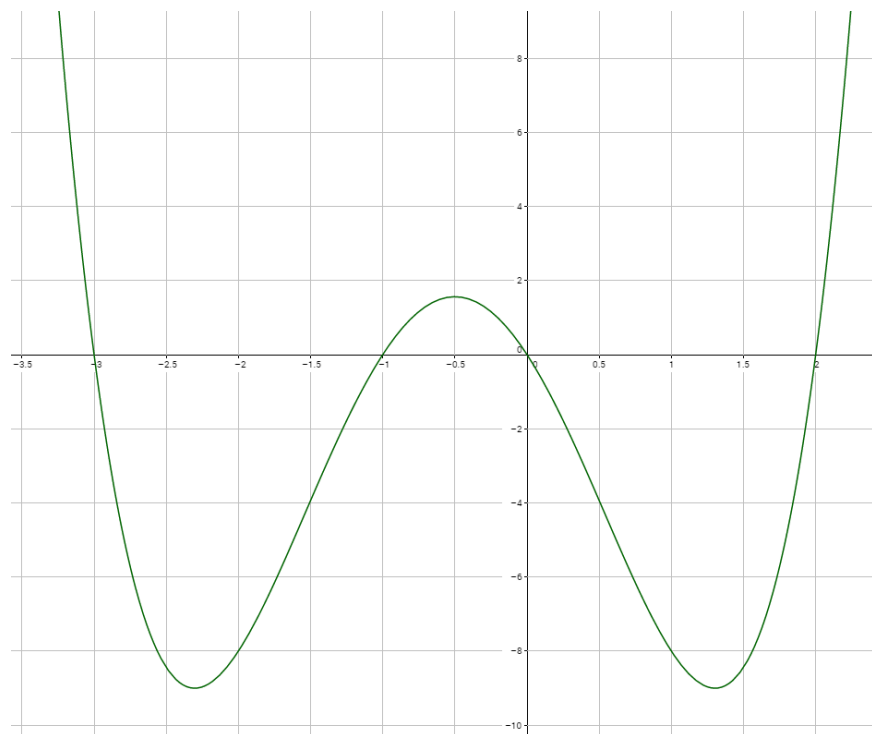
b. Approximate coordinates of local minimums

c. Interval(s) of increase

d. Interval(s) of decrease

* Answer each question regarding the characteristics of each function. Approximate all answers to the nearest hundredth.

2. $g(x) = x^4 + 2x^3 - 5x^2 - 6x$



a. Approximate coordinates of local maximums

b. Approximate coordinates of local minimums

c. Interval(s) of increase

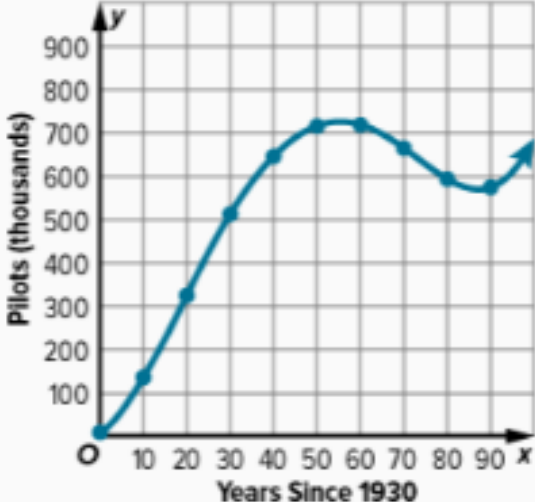
d. Interval(s) of decrease

Analyze a Polynomial Function



McGraw-Hill Education

PILOTS The total number of certified pilots in the United States is approximated by $f(x) = 0.0000903x^4 - 0.0166x^3 + 0.762x^2 + 6.317x + 7.708$, where x is the number of years after 1930 and $f(x)$ is the number of pilots in thousands. Graph the function and describe its key features over its relevant domain.



x	$f(x)$
0	7.708
10	131.381
20	320.496
30	507.961
40	648.356
50	717.933
60	714.616
70	658.001
80	589.356
90	571.621

Step 2 Describe the key features.

Domain and Range
The domain and range of the function is all real numbers. Because the function models years after 1930, the relevant domain and range are $\{x \mid x \geq 0\}$ and $\{f(x) \mid f(x) \geq 7.708\}$.

Extrema
There is a relative maximum between 1980 and 1990 and a relative minimum between 2010 and 2020 in the relevant domain.

End Behavior
As $x \rightarrow \infty, f(x) \rightarrow \infty$.

Intercepts
In the relevant domain, the y -intercept is at $(0, 7.708)$. There is no x -intercept, or zero, because the function begins at a value greater than 0 and as $x \rightarrow \infty, f(x) \rightarrow \infty$.

Symmetry
The graph of the function does not have symmetry.

Notes:

Copied from Module 4:2 Example 4.

McGraw-Hill Education

BACKPACKS The table shows U.S. backpack sales in millions of dollars, according to the Travel Goods Association. Make a scatter plot and a curve of best fit to show the trend over time. Then determine the backpack sales in 2015.

Year	Backpack Sales (million \$)	Year	Backpack Sales (million \$)
2000	1140	2008	1246
2001	1144	2009	1235
2002	1113	2010	1419
2003	1134	2011	1773
2004	1164	2012	1930
2005	1180	2013	2255
2006	1364	2014	2779
2007	1436		

Copied from Module 4:2 Extra Example 4.

 McGraw-Hill Education PROFIT The profit for each year since Sergio's Cycle Shop opened is shown in the table. Determine the polynomial function of best fit, where x represents the number of years since the shop has opened and y is profit. Then determine the estimated profit for Sergio's Cycle Shop in 2020.	<table border="1"> <thead> <tr> <th>Year</th><th>Profit (\$)</th></tr> </thead> <tbody> <tr><td>2010</td><td>80,700</td></tr> <tr><td>2011</td><td>72,100</td></tr> <tr><td>2012</td><td>74,300</td></tr> <tr><td>2013</td><td>74,000</td></tr> <tr><td>2014</td><td>76,200</td></tr> <tr><td>2015</td><td>84,500</td></tr> <tr><td>2016</td><td>92,300</td></tr> <tr><td>2017</td><td>106,400</td></tr> </tbody> </table>	Year	Profit (\$)	2010	80,700	2011	72,100	2012	74,300	2013	74,000	2014	76,200	2015	84,500	2016	92,300	2017	106,400
Year	Profit (\$)																		
2010	80,700																		
2011	72,100																		
2012	74,300																		
2013	74,000																		
2014	76,200																		
2015	84,500																		
2016	92,300																		
2017	106,400																		
Select the polynomial function of best fit, rounded to the nearest tenth. ☐ A $y = 3734.5x + 69,491.7$ ☐ B $y = 1468.5x^2 - 6544.6x + 79,770.8$ ☐ C $y = 2.3x^3 + 1444.6x^2 - 6482.1x + 79,745$ ☐ D $y = 45.3x^4 - 632.8x^3 + 4198.6x^2 - 10,201.7x + 80,291.3$																			
Use a graphing calculator to find the estimated profit in 2020 to the nearest dollar. \$																			
Select the assumption that was made to determine the profit in 2020. ☐ A The profit in 2010 was \$80,700. ☐ B A linear function does not model the data well. ☐ C The revenue continues to follow the polynomial trend. ☐ D The average rate of change is an increase of \$3671 each year.																			

Dividing by Monomials and Binomials (see also Mod 4: Lesson 4)

Copied from Module 4:4 Example 1. Find $(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$	Copied from Module 4:4 Extra Example 1. Divide a Polynomial by a Monomial Find $(28x^5y^4 + 12x^3y^3 - 44x^3y - 60x^2y) \div (4x^2y)$.
DIVIDING POLYNOMIALS USING LONG DIVISION Steps (ALGORITHM) for long division: 1) Divide; 2) Multiply; 3) Subtract; 4) Drop down the next digit.	
1. $267824 \div 76$	2. $111199 \div 431$
Copied from Module 4:4 Example 2. Divide a Polynomial by a Binomial Find $(x^2 - 5x - 36) \div (x + 4)$.	Copied from Module 4:4 Extra Example 2. Divide a Polynomial by a Binomial Find $(x^2 + 5x - 66)(x - 6)^{-1}$.
Copied from Module 4:4 Example 3. Find a Quotient with a Remainder Find $\frac{3z^3 - 14z^2 - 7z + 3}{z - 5}$. $z - 5 \overline{) 3z^3 - 14z^2 - 7z + 3}$	Copied from Module 4:4 Extra Example 3. Find a Quotient with a Remainder Select the quotient of $\frac{5x^4 + 7x^3 - 11x^2 - 7x + 8}{x - 1}$. ☐ A $5x^3 + 12x^2 + x - 4$ ☐ B $5x^3 + 2x^2 - 13x + 6 + \frac{2}{x-1}$ ☐ C $5x^3 + 7x^2 - 11x - 7 + \frac{8}{x-1}$ ☐ D $5x^3 + 12x^2 + x - 6 + \frac{2}{x-1}$

Extra Practice: Division of Polynomials (Module 4: Lesson 4)

3. $(x^3 - x^2 - 41x + 6) \div (x + 6)$	4. $(x^3 - 7x^2 + 6x + 9) \div (x - 7)$
5. $(x^4 + 2x^2 + 6) \div (x - 1)$	6. $(x^3 - 8x^2 + 13x + 26) \div (x^2 - 4x + 1)$

DIVIDING POLYNOMIALS USING SYNTHETIC DIVISION

Steps **(ALGORITHM)** for synthetic division: Drop down first value; then 1) Multiply; 2) Add;

7. $(x^3 - x^2 - 41x + 6) \div (x + 6)$	8. $(x^3 - 7x^2 + 6x + 9) \div (x - 7)$
9. $(x^4 + 2x^2 + 6) \div (x - 1)$	For more examples of Synthetic Division, please see Example 4 and Extra Example 4 in Module 4: Lesson 4.

BINOMIAL EXPANSION AND PASCAL'S TRIANGLE (Module 4: Lesson 5)

* Expand each of the following and write in standard form.

1. $(x + 1)^2$	2. $(x + 1)^3$
3. $(x + 1)^4$	4. $(x + 1)^5$
Pascal's Triangle	5. $(x + 1)^5$ Try this again using Pascal's Triangle.
6. $(x + 1)^7$ and this one.	
7. $(2x + y)^5$ and this one.	

Applications

8. When flipping 8 coins, what is the probability of flipping 3 Heads and 5 Tails?
9. As of 1/2/22, Donovan Mitchell has a 3-Point Percentage of .413. What is the probability that he will shoot 3 of 4 from behind the arc in his next game?

(Module 4: Lesson 5)

* Use your calculator and the graph of the function to determine each. Approximate all answers to the nearest hundredth.

1. $f(x) = -2x^3 + 4x^2 + x - 2$



a. $f(2) =$

b. $f(-1) =$

c. $f(0) =$

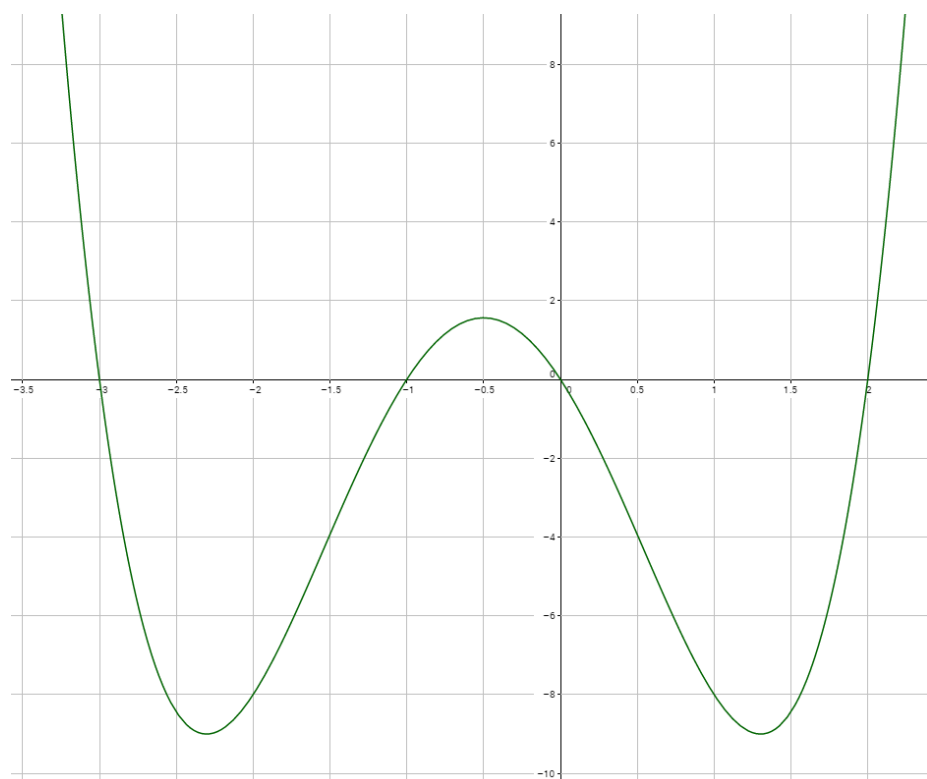
d. Determine the zeroes (roots) of $f(x)$.

e. What are the coordinates of the x -intercepts of the function?

f. Solve the equation
 $-2x^3 + 4x^2 + x - 2 = 0$

* Answer each question regarding the characteristics of each function. Approximate all answers to the nearest hundredth.

2. $g(x) = x^4 + 2x^3 - 5x^2 - 6x$



a. $f(-2) =$

b. $f(2) =$

c. $f(0) =$

d. Determine the zeroes (roots) of $f(x)$.

e. What are the coordinates of the x -intercepts of the function?

f. Solve the equation
 $-2x^3 + 4x^2 + x - 2 = 0$

FACTORING POLYNOMIAL EXPRESSIONS

Factoring is a tool used to change the form of an expression to assist in the solution process. We factor polynomials by using various factoring techniques.

Factoring Strategy:

1. Always look, first, for a common factor.

2. Then look at the number of terms.

Two terms: Determine whether you have a difference of squares (or sum/difference of cubes)

Three terms: Most likely a trinomial that you can factor as two quantities

Four terms: Try factoring by grouping

3. Always factor completely. Many times, more than one technique is used.

Formula for Sum/Difference of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

*Common Factor 1. $10x^2 - 40x$	*Sum of Cubes 5. $27a^3 + b^3$
*Factoring a Trinomial 2. $6x^2 + x - 12$	*Grouping 6. $x^3 - 3x^2 + 4x - 12$
*Difference of Squares 3. $4x^2 - 121$	*Mixed Techniques 1 7. $x^3 + 5x^2 - 4x - 20$
*Difference of Cubes 4. $x^3 - 8y^3$	*Mixed Techniques 2 8. $2x^5 - 20x^3 + 50x$

FACTORING PRACTICE

*Factor each completely.

9. $2x^2 - 128$	10. $a^2 - 10a + 25$	11. $4x^2 + 8x - 60$
12. $8x^3 - y^3$	13. $8y^2 - 18y - 5$	14. $x^2 + 4$
15. $x^3 + 7x - 3x^2 - 21$	16. $24x^2 - 54$	17. $x^4 - 2x^2 + 1$

MORE FACTORING PRACTICE

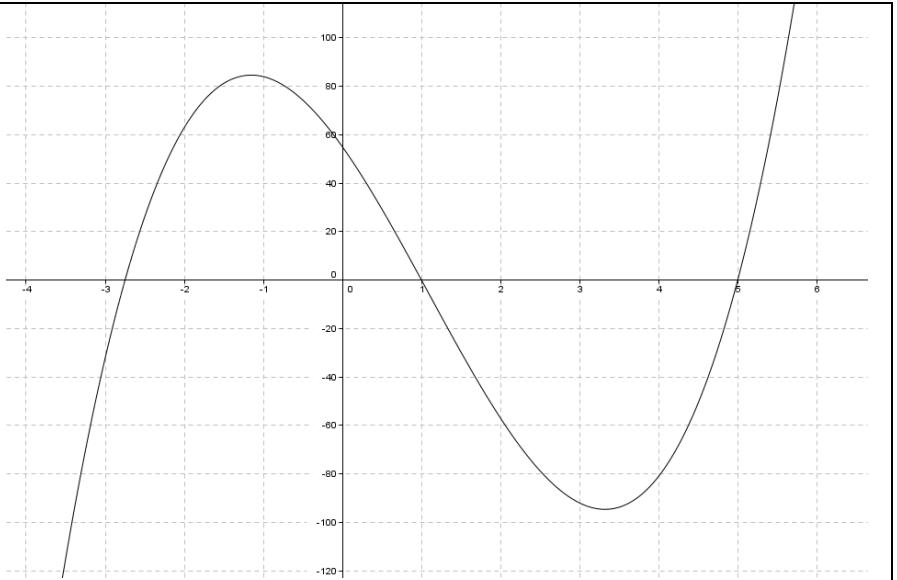
*Factor each completely.

18. $-12x^2 + 27$	19. $10a^2 - 7a - 12$	20. $4x^2 + 9x + 2$
21. $x^3 + 27$	22. $x^8 - y^4$	23. $27x^3 - 512$
24. $x^3 + 2x^2 - 6x - 12$	25. $2x^2 - x - 1$	26. $5x^4 + 13x^2 - 6$

Solving Polynomial Equations (Module 5: Lessons 1, 2, & 5)

1. No Calculators. Use the polynomial $y = 4x^3 - 13x^2 - 46x + 55$ to help determine the solutions for $4x^3 - 13x^2 - 46x + 55 = 0$.

x	y
-4	-225
-3	-32
-2	63
-1	84
0	55

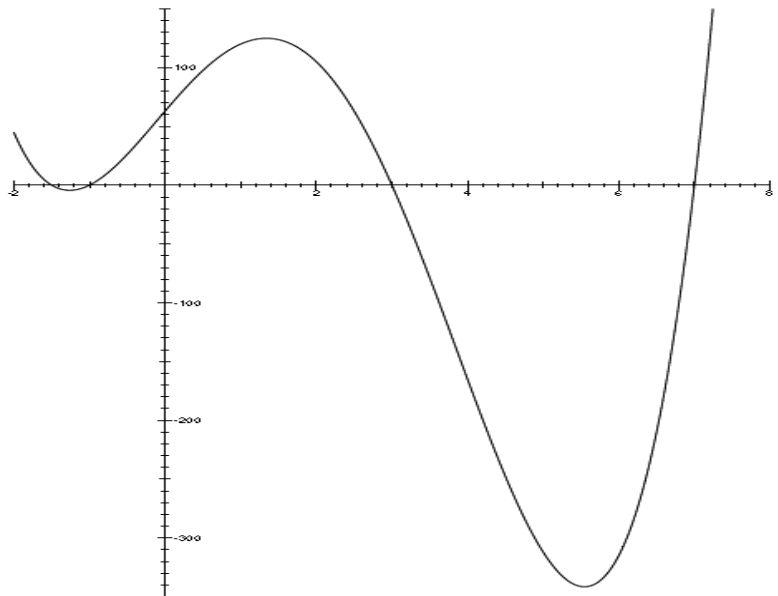


a. How many real solutions? _____

b. Using tables, graphs, and factoring determine all solutions.

2. No Calculators. Use the polynomial $y = 2x^4 - 15x^3 - 5x^2 + 75x + 63$ to help determine the solutions for $2x^4 - 15x^3 - 5x^2 + 75x + 63 = 0$.

x	y
-6	5265
-5	2688
-4	1155
-3	360
-2	45
-1	0



a. How many real solutions? _____

b. Using tables, graphs, and factoring determine all solutions.

Connection Between Factoring and Division

a. Completely Factor 30. b. Determine the value of each: $30 \div 30 =$ $30 \div 2 =$ $30 \div 5 =$ $30 \div 15 =$ $30 \div 6 =$	a. Completely Factor $x^2 - 3x - 28$. b. Determine the value of each: $(x^2 - 3x - 28) \div (x + 4) =$ $(x^2 - 3x - 28) \div (x - 7) =$ $(x^2 - 3x - 28) \div (x^2 - 3x - 28) =$ $(x^2 - 3x - 28) \div [(x + 4)(x - 7)] =$
--	---

*Solve each of the following using a method of your choice.

1. $x^3 - 5x^2 + 13x - 21 = 0$. If there are non-real solutions, determine the value of these as well.	2. $x^3 - 3x^2 + 4x - 12 = 0$ (Hint: some non-real solutions here)
3. $2x^3 + 9x^2 - 31x + 28 = 0$ (Hint: some non-real solutions here)	4. $6x^4 - 29x^3 + 30x^2 - 7x = 0$
5. 1. $x^2 - 4x - 21 = 0$	

Find the real zeroes (aka x-intercepts, aka roots) of each polynomial function.

1. $f(x) = (x - 1)(x + 4)(-x + 6)$	2. $x^3 - 3x^2 + 4x - 12 = y$
3. $f(x) = x^3 - 27$	4. $f(x) = x^4 - 2x^2 + 1$

*Investigating Multiplicity

Multiplicity considers the number of times that a factor is repeated in a function.

Sketch each example and discuss where x-intercepts occur and whether they turn or cross at the x-axis.

5. $f(x) = (x - 3)^2(x + 1)$	6. $f(x) = (x - 3)^2(x + 1)^4$	7. $f(x) = (x - 3)^3(x + 1)^2$
8. $f(x) = (x - 3)^2(x + 1)(x - 6)^4$	9. $f(x) = (x - 3)^3(x + 1)^4(x - 6)$	10. $f(x) = (x - 3)(x + 1)^3(x - 6)^5$

Talk about the multiplicity value and discuss polynomial functions with an even or odd degree.

Notes:

Make predictions about where and how x-intercepts occur on the following function.

$$f(x) = (x + 7)(5x + 2)^2(3x - 10)^3$$

WRITING POLYNOMIAL FUNCTIONS

*Given the zeroes (x-intercepts) of a polynomial function, write one possibility for this function in standard form.

1. Zeros: $-1, 2, 6$	2. Zeros: $2, \frac{1}{2}, -3$	3. Sketch the graph of a polynomial function with zeroes at $x = -1, 3, 5$ and a y-intercept at $(0, 15)$. Write the equation for this function that you have sketched first in factored form and then in standard form. Check your answer by graphing the function on your calculator.
----------------------	--------------------------------	--

