INTRODUCTION TO POLYNOMIALS

Patterns in Polynomials

* For each equation, fill in the table and then determine the pattern of the function values (aka y-values). Continue this process of finding patterns until you notice a constant pattern, i.e., a pattern with the same number repeated. You may need to work through this process two or three or more times.

1. $y = x^2$

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	49	36	25	16	9	4	1	0	1	4	9	16	25	36

-19 -9 -7 -5 -3 -1 +1 +3 +5

+2 +2 +2 +2 +2 +2 +2 +2

2. $y = 5x - 11$

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-46	-41	-36	-31	-26	-21	-16	-11	-6	-1	4	9	14	19

+5 +5 +5 +5 +5 +5 +5

3. $y = 2x^3 - 4x^2 + 5x - 7$

Use your graphing calculator to help you fill in this table.

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-924	-613	-382	-219	-112	-49	-18	-7	-4	3	26	77	168	518

+163 +107 +63 +31 +11 +3 +7 +23

-56 -44 -32 -20 -8 +4 +16

+12 +12 +12 +12 +12 +12

Classifying Polynomials *(see also Mod 4: Lesson 1)*

By Degree (the largest exponent)

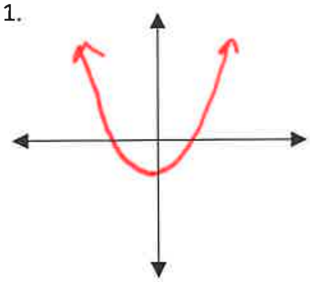
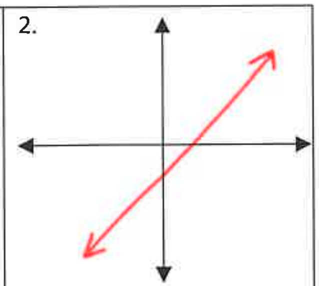
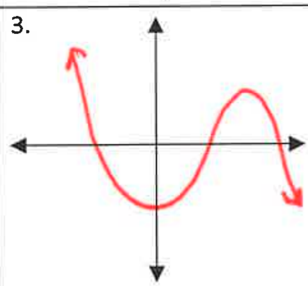
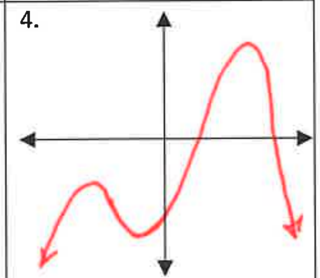
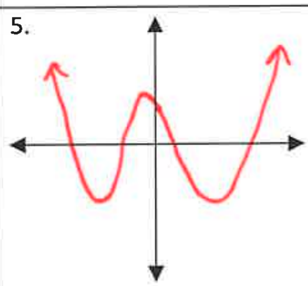
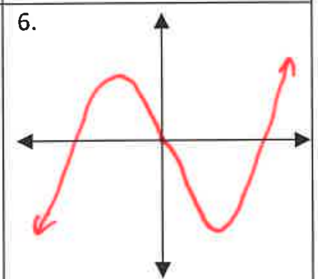
Degree	Classification
0	constant
1	linear or first degree
2	quadratic or second degree
3	cubic or third degree
4	fourth degree
5	5^{th} degree

By Number of Terms

Number of Terms	Classification
1	Monomial
2	Binomial
3	Trinomial
4	4 terms
5	5 terms

Rough Sketches

* Use your calculator to graph each function. Sketch the shape of each graph beside each function. Also, classify the polynomial by degree and number of terms as well as stating the leading coefficient.

1.a. $y = x^2 + x - 2$ QUADRATIC TRINOMIAL LEAD COEF. IS 1	1. 	2.a. $y = 2x - 5$ LINEAR BINOMIAL LEAD. COEF IS 2	2. 
3.a. $y = -2x^3 + 4x^2 + x - 2$ CUBIC POLYNOMIAL 4 TERMS LEAD COEF. IS -2	3. 	4.a. $y = -x^4 + 5x^3 + 5x^2 - x - 6$ 4TH DEGREE 5 TERMS LEAD. COEF. IS -1	4. 
5.a. $y = x^4 + 2x^3 - 5x^2 - 6x$ 4TH DEGREE 4 TERMS LEAD. COEF. IS 1	5. 	6.a. $y = -3x^2 + x^3 - 3x$ CUBIC TRINOMIAL LEAD COEF. IS 1	6. 

End Behavior *(see also Mod 4: Lesson 1)*

The **end behavior** of a polynomial **function** is the **behavior** of the graph of $f(x)$ as x approaches positive infinity or negative infinity. The degree and the leading coefficient of a polynomial **function** determines the **end behavior** of the graph.

To talk about end behavior, think about the direction the arrows would be pointing if you graphed each by hand.

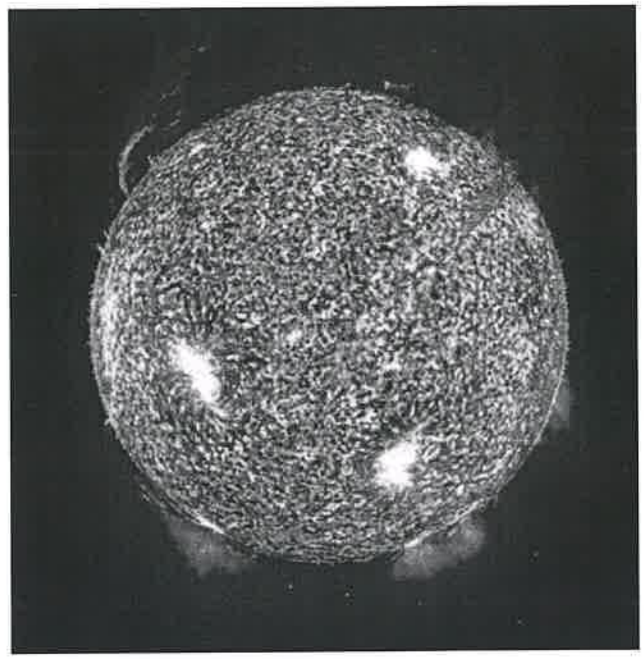
* Use your sketches to help fill in the table below. Think about what may cause turns and end behavior.

Equation	Degree	# of Turns	End Behavior
1.b. $y = x^2 + x - 2$	2	1	AS $x \to -\infty$, $y \to \infty$ AS $x \to \infty$, $y \to \infty$
2.b. $y = 2x - 5$	1	0	AS $x \to -\infty$, $y \to -\infty$ AS $x \to \infty$, $y \to \infty$
3.b. $y = -2x^3 + 4x^2 + x - 2$	3	2	AS $x \to -\infty$, $y \to \infty$, AS $x \to \infty$, $y \to -\infty$
4.b. $y = -x^4 + 5x^3 + 5x^2 - x - 6$	4	3	AS $x \to -\infty$, $y \to -\infty$ AS $x \to \infty$, $y \to -\infty$
5.b. $y = x^4 + 2x^3 - 5x^2 - 6x$	4	3	AS $x \to -\infty$, $y \to \infty$ AS $x \to \infty$, $y \to \infty$
6.b. $y = -3x^2 + x^3 - 3x$	3	2	AS $x \to -\infty$, $y \to -\infty$ AS $x \to \infty$, $y \to \infty$

Copied from Module 4:1 Example 4.

McGraw-Hill Education

SUN The density of the Sun, in grams per centimeter cubed, expressed as a percent of the distance from the core of the Sun to its surface can be modeled by the function $f(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$, where x represents the percent as a decimal. At the core $x = 0$, and at the surface $x = 1$.



CLASS DISCUSSION

Operations with Polynomials *(see also Mod 4: Lesson 3)*

* Write each sum or difference as a polynomial in standard form. Also, classify the polynomial by degree and number of terms as well as stating the leading coefficient.

1. $(x^3 + x^2 + x + 1) + (2x^3 + 3x^2 + x + 3)$ $3x^3 + 4x^2 + 2x + 4$ CUBIC POLYNOMIAL W/ 4 TERMS LEAD COEF. IS 3	2. $(1 - 5x + x^3) - (2x^4 + 5x^3 - 10x^2)$ $-2x^4 - 4x^3 + 10x^2 - 5x + 1$ 4TH DEGREE, 5 TERMS LEAD COEF. IS -2

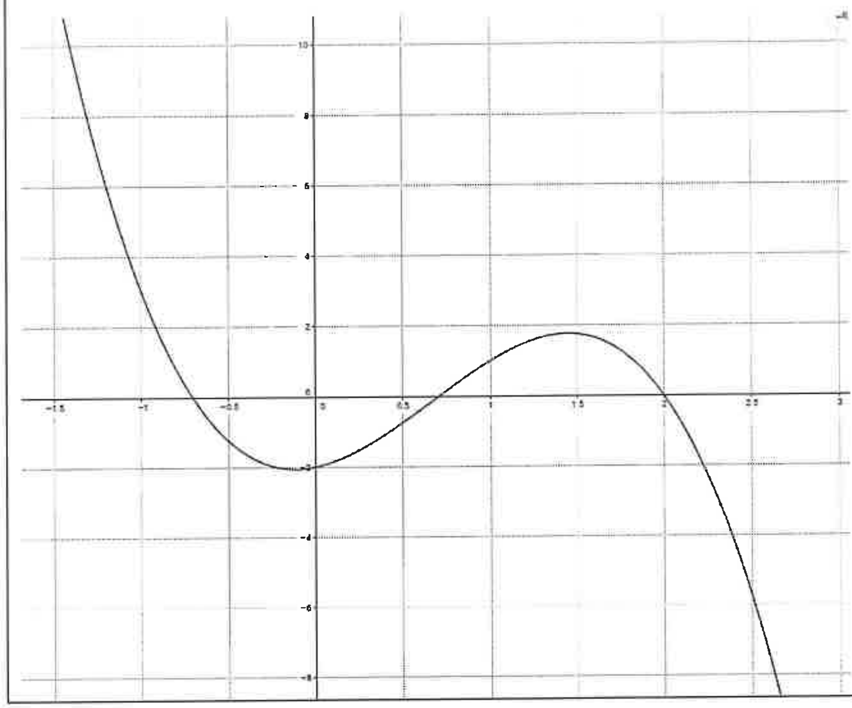
* Multiply each. Also, classify the polynomial by degree and number of terms as well as stating the leading coefficient.

3. $(x + 2)(x - 7)$ $x^2 - 5x - 14$ QUADRATIC TRINOMIAL LEAD COEF IS 1	4. $7x(x^2 - 3x + 1)$ $7x^3 - 21x^2 + 7x$ CUBIC TRINOMIAL LEAD COEF. IS 7	5. $(x + 2)(x^2 + 5)$ $x^3 + 2x^2 + 5x + 10$ CUBIC POLY W/ 4 TERMS LEAD COEF IS 1
6. $(x^2 - 2)(x^3 - x)$ $x^5 - 3x^3 + 2x$ 5TH DEGREE TRINOMIAL LEAD COEF. IS 1	7. $(x + 2)(x^2 - 3x + 1)$ $x^3 - x^2 - 5x + 2$ CUBIC 4 TERMS LEAD COEF IS 1	8. $(x^2 - 3x + 1)(2x^2 + x - 6)$ $2x^4 - 5x^3 - 7x^2 + 19x - 6$ 4TH DEGREE, 5 TERMS LEAD COEF IS 2

Extrema...Local Minima/Maxima *(see also Mod 4: Lesson 2)*

* Answer each question regarding the characteristics of each function. Approximate all answers to the nearest hundredth.

1. $f(x) = -2x^3 + 4x^2 + x - 2$



IN CLASS DISCUSSION

a. Approximate coordinates of local maximums

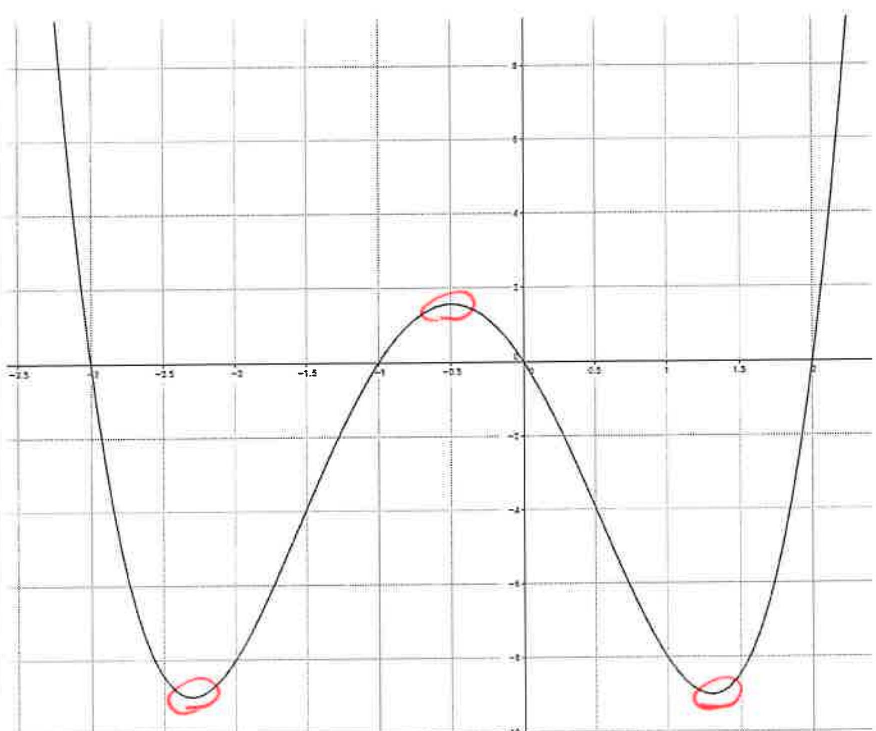
b. Approximate coordinates of local minimums

c. Interval(s) of increase

d. Interval(s) of decrease

* Answer each question regarding the characteristics of each function. Approximate all answers to the nearest hundredth.

2. $g(x) = x^4 + 2x^3 - 5x^2 - 6x$



a. Approximate coordinates of local maximums

(-.50, 1.75)

b. Approximate coordinates of local minimums

(-2.28, -8.90)

(1.33, -8.90)

c. Interval(s) of increase

(-2.28, -.50)

(1.33, ∞)

d. Interval(s) of decrease

(-∞, -2.28)

(-.50, 1.33)

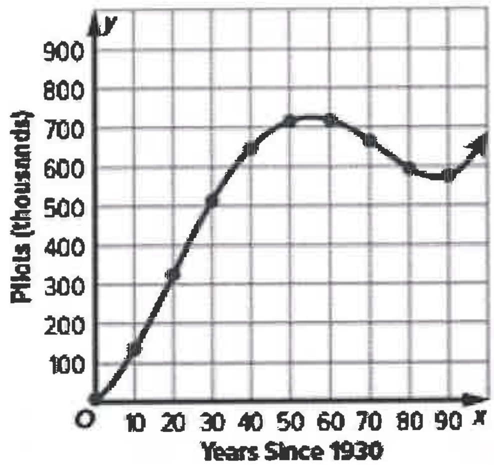
All was in class discussion

Copied from Module 4:2 Example 3.

Analyze a Polynomial Function

McGraw-Hill Education

PILOTS The total number of certified pilots in the United States is approximated by $f(x) = 0.0000903x^4 - 0.0166x^3 + 0.762x^2 + 6.317x + 7.708$, where x is the number of years after 1930 and $f(x)$ is the number of pilots in thousands. Graph the function and describe its key features over its relevant domain.

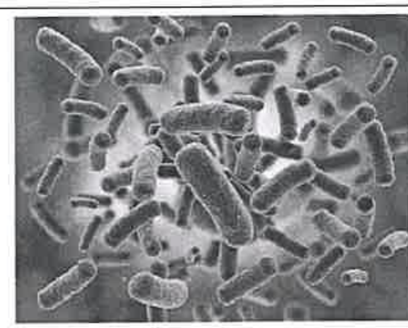


x	f(x)
0	7.708
10	131.381
20	320.496
30	507.961
40	648.356
50	717.933
60	714.616
70	658.001
80	589.356
90	571.621

Copied from Module 4:1 Extra Example 4.

BACTERIA The population of a bacteria sample $P(t)$ can be modeled by

$P(t) = 1000 - 19.75t + 20t^2 - \frac{1}{3}t^3$, where t is the time in days.

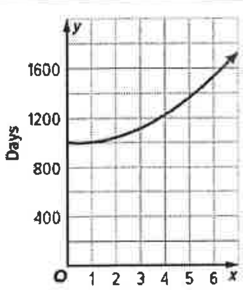


Part A
Find the population of bacteria after 3 days. Round to the nearest whole number.

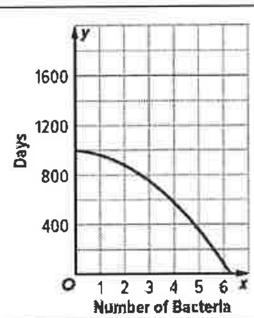
Part B

Select the graph of the number of bacteria over a week.

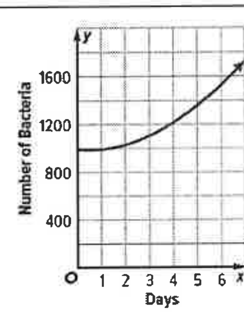
○ A



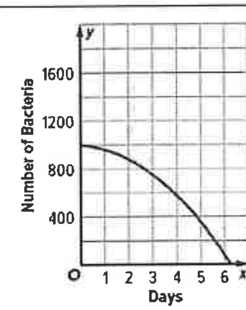
○ B



○ C



○ D



Dividing by Monomials and Binomials *(see also Mod 4: Lesson 4)*

Copied from Module 4:4 Example 1.

Find

$$(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$$

$$\frac{24a^4b^3}{6ab} + \frac{18a^2b^2}{6ab} - \frac{30ab^3}{6ab}$$

$$4a^3b^2 + 3ab - 5b^2$$

Copied from Module 4:4 Extra Example 1.

Divide a Polynomial by a Monomial

Find $(28x^5y^4 + 12x^3y^3 - 44x^3y - 60x^2y) \div (4x^2y)$.

$$\frac{28x^5y^4}{4x^2y} + \frac{12x^3y^3}{4x^2y} - \frac{44x^3y}{4x^2y} - \frac{60x^2y}{4x^2y}$$

$$7x^3y^3 + 3xy^2 - 11x - 15$$

DIVIDING POLYNOMIALS USING LONG DIVISION

Steps **(ALGORITHM)** for long division: 1) Divide; 2) Multiply; 3) Subtract; 4) Drop down the next digit.

1. 267824 ÷ 76

$$76\overline{)267824} = 3524 \quad R\,0$$

- 228
398
- 380
182
- 152
304
- 304
0

3524

2. 111199 ÷ 431

$$431\overline{)111199} = 258$$

- 862
2499
- 2155
3449
- 3448
1

$258\frac{1}{431}$

Copied from Module 4:4 Example 2.

Divide a Polynomial by a Binomial

Find $(x^2 - 5x - 36) \div (x + 4)$.

$$x+4\overline{)x^2 - 5x - 36} = x - 9$$

$-(x^2 + 4x)$

$-9x - 36$

$-(-9x - 36)$

0

$x - 9$

Copied from Module 4:4 Extra Example 2.

Divide a Polynomial by a Binomial

Find $(x^2 + 5x - 66)(x - 6)^{-1}$.

$$x-6\overline{)x^2 + 5x - 66} = x + 11$$

$-(x^2 - 6x)$

$11x - 66$

$-(11x - 66)$

0

$x + 11$

Copied from Module 4:4 Example 3.

Find a Quotient with a Remainder

Find $\frac{3z^3 - 14z^2 - 7z + 3}{z - 5}$.

$$z-5\overline{)3z^3 - 14z^2 - 7z + 3} = 3z^2 + z - 2$$

$-(3z^3 - 15z^2)$

$z^2 - 7z$

$-(z^2 - 5z)$

$-2z + 3$

Copied from Module 4:4 Extra Example 3.

Find a Quotient with a Remainder

Select the quotient of $\frac{5x^4 + 7x^3 - 11x^2 - 7x + 8}{x - 1}$.

- A $5x^3 + 12x^2 + x - 4$
- B $5x^3 + 2x^2 - 13x + 6 + \frac{2}{x-1}$
- C $5x^3 + 7x^2 - 11x - 7 + \frac{8}{x-1}$
- D $5x^3 + 12x^2 + x - 6 + \frac{2}{x-1}$

Extra Practice: Division of Polynomials (*Module 4: Lesson 4*)

3. $(x^3 - x^2 - 41x + 6) \div (x + 6)$

$$x+6\,\overline{\smash{)}\,x^3 - x^2 - 41x + 6}\quad \text{quotient: } x^2 - 7x + 1$$
$-(x^3 + 6x^2)$
$-7x^2 - 41x$
$-(-7x^2 - 42x)$
$x + 6$
$-(x + 6)$
0

$x^2 - 7x + 1$

4. $(x^3 - 7x^2 + 6x + 9) \div (x - 7)$

$$x-7\,\overline{\smash{)}\,x^3 - 7x^2 + 6x + 9}\quad \text{quotient: } x^2 + 6$$
$-(x^3 - 7x^2)$
$6x + 9$
$-(6x - 42)$
51

$x^2 + 6 + \frac{51}{x-7}$

5. $(x^4 + 2x^2 + 6) \div (x - 1)$

$$x-1\,\overline{\smash{)}\,x^4 + 0x^3 + 2x^2 + 0x + 6}\quad \text{quotient: } x^3 + x^2 + 3x + 3$$
$-(x^4 - x^3)$
$x^3 + 2x^2$
$-(x^3 - x^2)$
$3x^2 + 0x$
$-(3x^2 - 3x)$
$3x + 6$
$-(3x - 3)$
9

$x^3 + x^2 + 3x + 3 + \frac{9}{x-1}$

6. $(x^3 - 8x^2 + 13x + 26) \div (x^2 - 4x + 1)$

$$x^2-4x+1\,\overline{\smash{)}\,x^3 - 8x^2 + 13x + 26}\quad \text{quotient: } x - 4$$
$-(x^3 - 4x^2 + x)$
$-4x^2 + 12x + 26$
$-(-4x^2 + 16x - 4)$
$-4x + 30$

$x - 4 + \frac{-4x+30}{x^2-4x+1}$

DIVIDING POLYNOMIALS USING SYNTHETIC DIVISION

Steps **(ALGORITHM)** for synthetic division: Drop down first value; then 1) Multiply; 2) Add;

7. $(x^3 - x^2 - 41x + 6) \div (x + 6)$

-6	1	-1	-41	6
	↓	-6	42	-6
	1	-7	1	0

$x^2 - 7x + 1$

8. $(x^3 - 7x^2 + 6x + 9) \div (x - 7)$

7	1	-7	6	9
	↓	7	0	42
	1	0	6	51

$x^2 + 6 + \frac{51}{x-7}$

9. $(x^4 + 2x^2 + 6) \div (x - 1)$

1	1	0	2	0	6
	↓	1	1	3	3
	1	1	3	3	9

$x^3 + x^2 + 3x + 3 + \frac{9}{x-1}$

For more examples of Synthetic Division, please see Example 4 and Extra Example 4 in Module 4: Lesson 4.

BINOMIAL EXPANSION AND PASCAL'S TRIANGLE *(Module 4: Lesson 5)*

* Expand each of the following and write in standard form.

1. $(x+1)^2$ $= (x+1)(x+1)$

$= x^2 + 2x + 1$

2. $(x+1)^3$ $= (x^2 + 2x + 1)(x+1)$

$= x^3 + 3x^2 + 3x + 1$

3. $(x+1)^4$ $= (x^3 + 3x^2 + 3x + 1)(x+1)$

$= x^4 + 4x^3 + 6x^2 + 4x + 1$

4. $(x+1)^5$ $= (x^4 + 4x^3 + 6x^2 + 4x + 1)(x+1)$

$= x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

5. $(x+1)^5$ Try this again using Pascal's Triangle.

$= x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

6. $(x+1)^7$ and this one.

$= 1(x)^7(1)^0 + 7(x)^6(1)^1 + 21(x)^5(1)^2 + 35(x)^4(1)^3 + 35(x)^3(1)^4 + 21(x)^2(1)^5 + 7(x)^1(1)^6 + 1(x)^0(1)^7$

$= x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$

7. $(2x+y)^5$ and this one.

$= 1(2x)^5(y)^0 + 5(2x)^4(y)^1 + 10(2x)^3(y)^2 + 10(2x)^2(y)^3 + 5(2x)^1(y)^4 + 1(2x)^0(y)^5$

$32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$

Applications

8. When flipping 8 coins, what is the probability of flipping 3 Heads and 5 Tails?

$(\overset{T}{.5} + \overset{H}{.5})^8 = 1(\underset{T}{.5})^8(\underset{H}{.5})^0 + 8(\underset{T}{.5})^7(\underset{H}{.5})^1 + 28(\underset{T}{.5})^6(\underset{H}{.5})^2 + 56(\underset{T}{.5})^5(\underset{H}{.5})^3 + \ldots$

$\rightarrow$ ABOUT 22% CHANCE

9. As of 1/2/22, Donovan Mitchell has a 3-Point Percentage of .413. What is the probability that he will shoot 3 of 4 from behind the arc in his next game?

FACTORING POLYNOMIAL EXPRESSIONS

Factoring is a tool used to change the form of an expression to assist in the solution process. We factor polynomials by using various factoring techniques.

Factoring Strategy:

1. **Always look, first, for a common factor.**
2. **Then look at the number of terms.**
   - **Two terms: Determine whether you have a difference of squares (or sum/difference of cubes)**
   - **Three terms: Most likely a trinomial that you can factor as two quantities**
   - **Four terms: Try factoring by grouping**
3. **Always factor completely. Many times, more than one technique is used.**

Formula for Sum/Difference of Two Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

***Common Factor** 1. $10x^2 - 40x$ $10x(x-4)$	***Sum of Cubes** $(3a)^3 + (b)^3$ 5. $27a^3 + b^3$ $= (3a+b)(9a^2 - 3ab + b^2)$
***Factoring a Trinomial** 2. $6x^2 + x - 12$ $(3x-4)(2x+3)$	***Grouping** 6. $x^3 - 3x^2 + 4x - 12$ $x^2(x-3) + 4(x-3)$ $(x-3)(x^2+4)$
***Difference of Squares** 3. $4x^2 - 121$ $(2x+11)(2x-11)$	*** Mixed Techniques 1** 7. $x^3 + 5x^2 - 4x - 20$ $x^2(x+5) - 4(x+5)$ $(x+5)(x^2-4)$ $(x+5)(x+2)(x-2)$
***Difference of Cubes** $(x)^3 - (2y)^3$ 4. $x^3 - 8y^3$ $= (x-2y)(x^2 + 2xy + 4y^2)$	***Mixed Techniques 2** 8. $2x^5 - 20x^3 + 50x$ $2x(x^4 - 10x^2 + 25)$ $2x(x^2-5)(x^2-5)$ $2x(x^2-5)^2$

FACTORING PRACTICE

*Factor each completely.

9. $2x^2 - 128$ $2(x^2 - 64)$ $2(x+8)(x-8)$	10. $a^2 - 10a + 25$ $(a-5)(a-5)$ or $(a-5)^2$	11. $4x^2 + 8x - 60$ $4(x^2 + 2x - 15)$ $4(x+5)(x-3)$
12. $8x^3 - y^3$ $\quad (2x)^3 - (y)^3$ $= (2x-y)(4x^2 + 2xy + y^2)$	13. $8y^2 - 18y - 5$ $(4y+1)(2y-5)$	14. $x^2 + 4$ DOESN'T FACTOR PRIME
15. $\underbrace{x^3 + 7x}\underbrace{- 3x^2 - 21}$ $x(x^2+7) - 3(x^2+7)$ $(x^2+7)(x-3)$	16. $24x^2 - 54$ $6(4x^2 - 9)$ $6(2x+3)(2x-3)$	17. $x^4 - 2x^2 + 1$ $(x^2 - 1)(x^2 - 1)$ $(x+1)(x-1)(x+1)(x-1)$

MORE FACTORING PRACTICE

*Factor each completely.

18. $-12x^2 + 27$ $-3(4x^2 - 9)$ $-3(2x+3)(2x-3)$	19. $10a^2 - 7a - 12$ $(5a+4)(2a-3)$	20. $4x^2 + 9x + 2$ $(4x+1)(x+2)$
21. $x^3 + 27$ $\quad (x)^3 + (3)^3$ $= (x+3)(x^2 - 3x + 9)$	22. $x^8 - y^4$ $(x^4 + y^2)(x^4 - y^2)$ $(x^4+y^2)(x^2+y)(x^2-y)$	23. $27x^3 - 512$ $\quad (3x)^3 - (8)^3$ $= (3x-8)(9x^2 + 24x + 64)$
24. $\underbrace{x^3 + 2x^2}\underbrace{- 6x - 12}$ $x^2(x+2) - 6(x+2)$ $(x+2)(x^2-6)$	25. $2x^2 - x - 1$ $(2x+1)(x-1)$	26. $5x^4 + 13x^2 - 6$ $(5x^2 - 2)(x^2 + 3)$